NACA TN 2590

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2590

CALCULATIONS ON THE FORCES AND MOMENTS FOR AN OSCILLATING WING-AILERON COMBINATION IN TWO-DIMENSIONAL POTENTIAL FLOW AT SONIC SPEED

By Herbert C. Nelson and Julian H. Berman

Langley Aeronautical Laboratory
Langley Field, Va.

Washington
January 1952

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SUMMARY

The linearized theory for compressible unsteady flow is used, as suggested in recent contributions to the subject, to obtain the velocity potential and the lift and moment for a thin, harmonically oscillating, two-dimensional wing-aileron combination moving at sonic speed. The velocity potential is derived by considering the sonic case as the limit of the linearized supersonic theory. From the velocity potential explicit expressions for the lift and moment are developed for vertical translation and pitching of the wing and rotation of the aileron. The paper provides extensive tables of numerical values for the coefficients contained in the expressions for lift and moment, for various values of the reduced frequency k $(0 < k \leq 3.5)$ and aileron hinge position (from 10 to 90 percent of the wing chord). The sonic results are compared and found to be consistent with previously obtained subsonic and supersonic results. Several figures are presented showing the variation of lift and moment with reduced frequency and Mach number and the influence of Mach number on some cases of bending-torsion flutter.

INTRODUCTION

Instability investigations for high-speed aircraft often require a knowledge of the air forces and moments that act on an oscillating wing moving at high speed. For subsonic and supersonic speeds the main source of theoretical information has been the solution of the linearized differential equation for compressible flow. For sonic or near-sonic speed, however, the linearized theory has been generally assumed inapplicable, since it does not allow for thickness effects, shocks, and strong disturbances. As is well known, it predicts infinite forces on a nonoscillating, thin, unswept wing moving at sonic speed.

Important differences exist, however, between the steady and unsteady cases. By a discussion of the order of magnitude of the terms of the general nonlinear differential equation for compressible flow, reference 1 shows that for unsteady two-dimensional flow at sonic speed this equation is essentially linear and in linear form leads to physically plausible results for the forces on a thin oscillating wing, provided the frequency of oscillation is sufficiently large. A similar conclusion was reached in reference 2, where linear methods applied to a wing in two-dimensional nonstationary flow at sonic speed yielded perturbation velocities of the same order of magnitude as those obtained for subsonic or supersonic speeds. In references 3 and 4 expressions and some numerical values are given for the lift forces and moments on an oscillating two-dimensional wing moving at sonic speed. Because of the importance of the sonic problem in present-day flight considerations and because of the insight into the three-dimensional problem that the solution for two-dimensional flow will probably afford, the purpose of the present paper is to develop the two-dimensional case more fully.

Consideration is thus given to the case of an oscillating wing-aileron combination in two-dimensional flow at sonic speed. The velocity potential for this case is obtained, and from the velocity potential expressions for the air forces and moments on the wing-aileron combination are developed in terms of the frequency of oscillation. Numerical tables of the coefficients contained in the expressions for lift and moment are supplied which may be used for the theoretical calculations involved in wing flutter and other instability problems for sonic speed. The tables provide a means for obtaining continuity of calculation between high-subsonic and low-supersonic results for the oscillating wing-aileron combination in two-dimensional flow.

Because of the small-disturbance assumption, the theory and subsequent results are subject to the same restrictions imposed on all small-perturbation theory, subsonic and supersonic. In addition, as the frequency approaches zero, the difficulties of the steady linearized problem are encountered; therefore the validity of the subsequent results is subject to question for the range of low frequencies. Moreover, uncertainty exists because the linear unsteady results are considered to represent disturbances from an equilibrium position that is governed by nonlinear relations, and a great amount of experimentation may be necessary to determine the region of validity for the calculations. Nevertheless, the results serve as a bridge between subsonic and supersonic theory and may be applicable for a range of high frequencies.

SYMBOLS

a	velocity of sound in undisturbed medium
b	wing semichord
c_l	section lift coefficient
c_m	section moment coefficient about leading edge
$f(r_j)$	Fresnel integral defined in equation (23)
h	vertical displacement of axis of rotation
$J_0(\lambda)$	Bessel function of zero order (first kind)
k	reduced frequency $(\omega b/V)$
$k' = \dfrac{\omega b}{a}$	
L_i,M_i,N_i	quantities defined by equation (22); $i = 1, 2, 3, 4, 5,$ and 6
L_i',M_i',N_i'	quantities defined by equation (23); independent of wing-axis-of-rotation position
m	mass of wing per unit span
M	Mach number (V/a)
M_α	aerodynamic section moment on wing about axis of rotation, positive leading edge up
M_β	aerodynamic section moment on aileron about its hinge, positive leading edge up
Δp	pressure difference
P	aerodynamic section normal force, positive downward
t	time
V	flight speed
$w(2bx,t)$	normal velocity at x, at time t

x, z	nondimensional rectangular coordinates, referred to $2b$
$x' = 2bx$	
$y' = 2by$	
$z' = 2bz$	
x_0	abscissa of axis of rotation of wing section; referred to $2b$
x_1	abscissa of aileron hinge; referred to $2b$
x_α	location of center of gravity of wing-aileron system measured from elastic axis (see reference 5)
α	angular displacement (pitch) about axis of rotation
α_h	effective angle of attack due to vertical translation $(\dot{h}/V)$
β	angular displacement of aileron; measured relative to α
θ_h	phase angle between lift due to h and bending velocity $\dot{h}$
θ_α	phase angle between lift due to α and position α
θ_{hm}	phase angle between moment due to h and bending velocity $\dot{h}$
$\theta_{\alpha m}$	phase angle between moment due to α and position α
κ	density parameter $\left(\frac{\pi\rho b^2}{m}\right)$; reference 5 uses $\mu = \frac{\pi}{4}\frac{1}{\kappa}$
ξ	abscissa of point of disturbance; referred to $2b$
$\xi' = 2b\xi$	
ρ	density in main stream
τ	time variable
τ_1, τ_2	times required for transmittal of disturbance to field point
ϕ	disturbance velocity potential
ω	angular frequency of oscillation

ω_h natural bending frequency of wing

ω_α natural torsional frequency of wing

ANALYSIS

The theory presented herein for two-dimensional flow at sonic speed is based on the assumptions that the two wing surfaces act independently and that wake effects are absent. Thus the sonic case as treated is more akin to the supersonic than the subsonic case. The velocity potential for the oscillating two-dimensional wing moving at sonic speed is derived by allowing the Mach number M to approach unity in the velocity potential for the wing moving at supersonic speed. An alternative derivation is also given in which the potential is obtained directly from the linearized differential equation by a method of solution employing the Laplace transformation. In reference 3 Rott obtained the velocity potential by superposition of the elementary source solution of the linearized differential equation.

<u>Velocity potential for sonic speed</u>.- Consider first the velocity potential for a harmonically oscillating two-dimensional wing moving at supersonic speed, given in reference 5 as

$$\phi(2bx,t) = -\frac{2b}{\pi\sqrt{M^2 - 1}}\int_0^x\int_{\tau_1}^{\tau_2}\frac{w(2b\xi,t)e^{-i\omega\tau}\,d\tau\,d\xi}{\sqrt{(\tau - \tau_1)(\tau_2 - \tau)}} \tag{1}$$

where

$$\tau_1 = \frac{2b(x - \xi)}{a(M + 1)}$$

$$\tau_2 = \frac{2b(x - \xi)}{a(M - 1)}$$

a is the speed of sound in the undisturbed medium, x and ξ are nondimensional coordinates referred to the wing chord $2b$, $w(2b\xi,t)$ is the prescribed local normal velocity at the wing surface, and ω is the frequency of oscillation. The integral in equation (1) represents the total effect of all the disturbances created by the wing. The time-lag functions τ_1 and τ_2 are associated with the two pulses that occur at the point x because of a disturbance created at the point ξ (see

reference 5 for more complete discussion). Another form for equation (1), also given in reference 5, is

$$\phi(2bx,t) = -\frac{2b}{\sqrt{M^2 - 1}} \int_0^x w(2b\xi,t)e^{-i2k'\frac{M(x-\xi)}{M^2-1}} J_0\left(2k' \frac{x - \xi}{M^2 - 1}\right)d\xi \tag{2}$$

where $k' = \frac{b\omega}{a}$.

As the Mach number M approaches unity the argument of the Bessel function J_0 in equation (2) becomes infinite, and the following asymptotic approximation is applicable:

$$\lim_{M\to 1} J_0\left(2k' \frac{x - \xi}{M^2 - 1}\right) = \sqrt{\frac{M^2 - 1}{\pi k(x - \xi)}} \cos\left(2k \frac{x - \xi}{M^2 - 1} - \frac{\pi}{4}\right) \tag{3}$$

where on the right-hand side k' has been replaced by k since $k = \frac{b\omega}{V}$ and $k' = \lim_{M\to 1} k$. At $M = 1$ the time-lag function τ_2 contained in equation (1) becomes infinite and the influence of one of the two pulses characteristic of supersonic flow becomes vanishingly small. (By considering the sonic case as a limit from the subsonic side, the wing at sonic speed cannot overtake the second pulse.)

By letting M approach unity in equation (2) and using equation (3) in the process, the sonic velocity potential is found to be

$$\phi(2bx,t) = -2b \int_0^x w(2b\xi,t)G(x - \xi)d\xi \tag{4}$$

where

$$G(x) = \frac{1}{2} \frac{e^{-ikx}}{\sqrt{i\pi kx}}$$

Equation (4) can also be obtained directly from the linearized differential equation for two-dimensional compressible flow, which may be written as

$$\frac{1}{a^2}\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x'}\right)^2 \phi = \frac{\partial^2\phi}{\partial x'^2} + \frac{\partial^2\phi}{\partial z'^2} , \qquad (5)$$

For the harmonically oscillating wing moving at sonic speed, equation (5) becomes

$$\frac{\partial^2\psi}{\partial z'^2} = -\frac{\omega^2}{a^2}\psi + \frac{2i\omega}{a}\frac{\partial\psi}{\partial x'} \qquad (6)$$

where the disturbance velocity potential ϕ is related to ψ by

$$\phi(x',z',t) = \psi(x',z')e^{i\omega t}$$

and $x' = 2bx$ and $z' = 2bz$. The mean position of the wing (given by $z' = 0$ and $x' \geqq 0$) and the rectangular coordinate system being used are moving at velocity $V = a$ in the negative x'-direction, as shown in figure 1. Since this paper is concerned only with the lift of a thin wing, the boundary conditions that equation (6) must satisfy are

$$\left(\frac{\partial\psi}{\partial z'}\right)_{z'=\pm 0} = w(x') \qquad (x' \geqq 0) \quad (7)$$

$$\psi \rightarrow 0 \quad \text{as} \quad z' \rightarrow \pm\infty \qquad (8)$$

$$\psi = 0 \qquad (x' < 0) \quad (9)$$

In accordance with small-disturbance linearized theory the boundary conditions are expressed for the mean position of the wing rather than the wing itself. Equation (7) implies that the normal-velocity distribution on the wing is given; equation (8) is a condition on the behavior at infinity (the manner of approaching zero is associated with the radiation condition of Sommerfeld); equation (9) is the condition that no disturbances be propagated forward of the wing. Since the velocity potential must be continuous, equation (9) implies that

$$\psi(+0,z') = \psi(-0,z') = 0$$

Equations (6) to (9) constitute the boundary-value problem for the velocity potential ϕ.

As a matter of possible interest an alternative derivation of equation (4) that makes use of the Laplace transformation (as was done by Stewartson in reference 6 for supersonic flow) is presented. Applying the Laplace transform

$$\bar{\psi}(s,z') = \int_0^\infty e^{-sx'}\psi(x',z')dx'$$

to equations (6) to (9) yields

$$\left(\frac{d^2\bar{\psi}}{dz'^2}\right) = \left(\frac{2i\omega}{a}s - \frac{\omega^2}{a^2}\right)\bar{\psi} \equiv \mu^2\bar{\psi} \tag{10}$$

$$\left(\frac{d\bar{\psi}}{dz'}\right)_{z'=\pm 0} = w(s) \tag{11}$$

$$\bar{\psi} \longrightarrow 0 \quad \text{as} \quad z' \longrightarrow \pm\infty \tag{12}$$

From equations (10) to (12) the value for $\bar{\psi}$ is

$$\bar{\psi} = -\frac{z'}{|z'|}\frac{w(s)e^{-\mu z'}}{\mu} \tag{13}$$

From equation (13) the value for $\bar{\psi}$ at the upper surface of the wing $(z' = +0)$ is

$$\bar{\psi} = -\frac{w(s)}{\mu} \tag{14}$$

Applying the inverse transform to equation (14) yields

$$\psi(x',+0) = -\int_0^{x'} w(\xi')G(x' - \xi')d\xi'$$

or

$$\phi(x',+0,t) = -e^{i\omega t}\int_0^{x'} w(\xi')G(x' - \xi')d\xi' \tag{15}$$

where

$$G(x') = \frac{1}{2} \frac{e^{-\frac{i\omega}{2a}x'}}{\sqrt{\pi \cdot \frac{i\omega}{2a} x'}}$$

$$\xi' = 2b\xi$$

Equations (15) and (4) are identical, each giving the velocity potential at the upper surface of the wing.

Application to wing-aileron combination.- For the particular case of the wing-aileron combination oscillating harmonically in vertical translation h, pitch α, and aileron rotation β (see fig. 1(b)), the normal velocity at a point x of the wing chord may be expressed as

$$w(2bx,t) = -\left[\dot{h} + V\alpha + 2b(x - x_0)\dot{\alpha} + V\beta + 2b(x - x_1)\dot{\beta}\right] \tag{16}$$

where

$$h = h_0 e^{i\omega t}$$

$$\alpha = \alpha_0 e^{i\omega t}$$

$$\beta = \beta_0 e^{i\omega t}$$

h_0, α_0, and β_0 are complex amplitudes, and the β terms are to be interpreted as zero for $x < x_1$. Since linearized theory is being employed, the potential given in equation (4) may be considered as the sum of five potentials, each of which is associated with one of the terms on the right-hand side of equation (16). Hence the potential may be written as

$$\phi = \phi_{\dot{h}} + \phi_{\alpha} + \phi_{\dot{\alpha}} + \phi_{\beta} + \phi_{\dot{\beta}} \tag{17}$$

where upon substituting equation (16) into equation (4)

$$\phi_{\dot{h}} = 2b\dot{h} \int_0^x G(x - \xi)d\xi$$

$$\phi_{\alpha} = 2bV\alpha \int_0^x G(x - \xi)d\xi$$

$$\phi_{\dot{\alpha}} = 4b^2\dot{\alpha} \int_0^x (\xi - x_0)G(x - \xi)d\xi$$

$$\phi_{\beta} = 2bV\beta \int_{x_1}^x G(x - \xi)d\xi$$

$$\phi_{\dot{\beta}} = 4b^2\dot{\beta} \int_{x_1}^x (\xi - x_1)G(x - \xi)d\xi$$

Forces and moments.- The velocity potential for the upper wing surface given in equation (4) is antisymmetric with respect to the plane $z = 0$, as may be noted in the boundary condition, equation (7). The local pressure difference, positive downward, between the upper and lower surfaces of the wing at any point x is obtained from equation (4) by means of

$$\Delta p = -2\rho\left(\frac{\partial\phi}{\partial t} + \frac{V}{2b}\frac{\partial\phi}{\partial x}\right)$$

where ρ is the density of the undisturbed medium. The force (positive downward) acting on a wing section is therefore

$$P = 2b \int_0^1 \Delta p \, dx \tag{18}$$

The moments (positive leading edge up) acting on the entire wing section about the axis of rotation at x_0 and on the aileron section about the hinge point x_1 are, respectively,

$$M_\alpha = 4b^2 \int_0^1 (x - x_0)\Delta p \, dx \tag{19}$$

$$M_\beta = 4b^2 \int_{x_1}^1 (x - x_1)\Delta p \, dx \tag{20}$$

Upon substituting equation (17) into equations (18), (19), and (20) and performing the indicated integrations, the results may be written as

$$\left.\begin{aligned}
P &= -4\rho bV^2k^2e^{i\omega t}\left[\frac{h_0}{b}\left(L_1 + iL_2\right) + \alpha_0\left(L_3 + iL_4\right) + \beta_0\left(L_5 + iL_6\right)\right] \\
M_\alpha &= -4\rho b^2V^2k^2e^{i\omega t}\left[\frac{h_0}{b}\left(M_1 + iM_2\right) + \alpha_0\left(M_3 + iM_4\right) + \beta_0\left(M_5 + iM_6\right)\right] \\
M_\beta &= -4\rho b^2V^2k^2e^{i\omega t}\left[\frac{h_0}{b}\left(N_1 + iN_2\right) + \alpha_0\left(N_3 + iN_4\right) + \beta_0\left(N_5 + iN_6\right)\right]
\end{aligned}\right\} \tag{21}$$

The coefficients of equation (21) can be expressed as follows with primed quantities introduced for convenience in numerical tabulation to denote terms independent of the wing-axis-of-rotation position x_0 (referred to $x_0 = 0$):

$$\left.\begin{aligned}
L_1 + iL_2 &= L_1' + iL_2' \\
L_3 + iL_4 &= L_3' + iL_4' - 2x_0\left(L_1' + iL_2'\right) \\
L_5 + iL_6 &= L_5' + iL_6' \\
M_1 + iM_2 &= M_1' + iM_2' - 2x_0\left(L_1' + iL_2'\right) \\
M_3 + iM_4 &= M_3' + iM_4' - 2x_0\left[\left(M_1' + iM_2'\right) + \left(L_3' + iL_4'\right)\right] + {} \\
&\qquad 4x_0^2\left(L_1' + iL_2'\right) \\
M_5 + iM_6 &= N_5' + iN_6' + 2\left(x_1 - x_0\right)\left(L_5' + iL_6'\right) \\
N_1 + iN_2 &= N_1' + iN_2' + M_1' + iM_2' - 2x_1\left(L_1' + iL_2'\right) \\
N_3 + iN_4 &= N_3' + iN_4' - 2x_0\left(N_1 + iN_2\right) \\
N_5 + iN_6 &= N_5' + iN_6'
\end{aligned}\right\} \tag{22}$$

The primed quantities, as a result of integration by parts, can be expressed as

$$
\left.
\begin{aligned}
L_1' + iL_2' &= -\frac{1 - i}{r_0} f\left(r_0\right) + \frac{1 + i}{r_0^2}\sqrt{\frac{r_0}{2\pi}}\, e^{-ir_0} \\
L_3' + iL_4' &= \frac{1 - i}{2r_0}\left(-2 + \frac{2i}{r_0} + \frac{1}{2r_0^2}\right) f\left(r_0\right) + \frac{1 + i}{2r_0^2}\sqrt{\frac{r_0}{2\pi}}\, e^{-ir_0}\left(2 - \frac{i}{r_0}\right) \\
L_5' + iL_6' &= \left(1 - x_1\right)^3\left[\frac{1 - i}{2r_1}\left(-2 + \frac{2i}{r_1} + \frac{1}{2r_1^2}\right) f\left(r_1\right) + {}\right. \\
&\qquad \left.\frac{1 + i}{2r_1^2}\sqrt{\frac{r_1}{2\pi}}\, e^{-ir_1}\left(2 - \frac{i}{r_1}\right)\right] \\
M_1' + iM_2' &= \frac{1 - i}{2r_0}\left(-2 - \frac{1}{2r_0^2}\right) f\left(r_0\right) + \frac{1 + i}{2r_0^2}\sqrt{\frac{r_0}{2\pi}}\, e^{-ir_0}\left(2 - \frac{i}{r_0}\right) \\
M_3' + iM_4' &= \frac{1 - i}{2r_0}\left(-\frac{8}{3} + \frac{2i}{r_0} - \frac{i}{2r_0^3}\right) f\left(r_0\right) + \frac{1 + i}{2r_0^2}\sqrt{\frac{r_0}{2\pi}}\, e^{-ir_0}\left(\frac{8}{3} - \frac{2i}{3r_0} + \frac{1}{r_0^2}\right) \\
N_1' + iN_2' &= x_1^3\left[\frac{1 - i}{2r_2}\left(-2 + \frac{1}{2r_2^2}\right) f\left(r_2\right) + \frac{1 + i}{2r_2^2}\sqrt{\frac{r_2}{2\pi}}\, e^{-ir_2}\left(2 + \frac{i}{r_2}\right)\right] \\
N_3' + iN_4' &= x_1^4\left[\frac{1 - i}{2r_2}\left(-\frac{4}{3} + \frac{2i}{r_2} + \frac{1}{r_2^2} + \frac{i}{2r_2^3}\right) f\left(r_2\right) + \frac{1 + i}{2r_2^2}\sqrt{\frac{r_2}{2\pi}}\left(\frac{4}{3} - \frac{4i}{3r_2} - {}\right.\right. \\
&\qquad \left.\left.\frac{1}{r_2^2}\right) e^{-ir_2}\right] + M_3' + iM_4' - 2x_1\left(L_3' + iL_4'\right) \\
N_5' + iN_6' &= \left(1 - x_1\right)^4\left[\frac{1 - i}{2r_1}\left(-\frac{8}{3} + \frac{2i}{r_1} - \frac{i}{2r_1^3}\right) f\left(r_1\right) + {}\right. \\
&\qquad \left.\frac{1 + i}{2r_1^2}\sqrt{\frac{r_1}{2\pi}}\left(\frac{8}{3} - \frac{2i}{3r_1} + \frac{1}{r_1^2}\right) e^{-ir_1}\right]
\end{aligned}
\right\}
\quad (23)
$$

where

$$r_0 = k$$

$$r_1 = \left(1 - x_1\right)k$$

$$r_2 = x_1 k$$

and the quantities $f\left(r_j\right)$ are the Fresnel integrals

$$f\left(r_j\right) = \int_0^{r_j} \frac{e^{-ix}}{\sqrt{2\pi x}}\, dx \qquad (j = 0, 1, 2)$$

The primed quantities L_i' and M_i' $(i = 1, 2, 3, \text{and } 4)$, associated with wing bending torsion, are tabulated in table I as functions of the reduced frequency k for the range $0 < k \leqq 3.5$. The primed quantities L_i', M_i' $(i = 5 \text{ and } 6)$, and N_i' $(i = 1, 2, 3, 4, 5, \text{and } 6)$, introduced by the aileron degree of freedom, are tabulated in or can be obtained from table II for the same values of k and for values of the aileron hinge position x_1 ranging from 0.1 to 0.9 in increments of 0.1. In order to make the tabulated values more uniform, each of the primed quantities listed in the tables has been multiplied by the reduced frequency squared k^2, which appears in the force and moment equations, equations (21).

DISCUSSION

Lift forces and moments.- The lift forces and moments, the coefficients of which are given in table I, apply to a thin, oscillating, two-dimensional wing moving at sonic speed. A comparison of these results with the forces and moments previously obtained for the same type of wing moving at subsonic and supersonic speeds (references 5, 7, 8, and other papers) may be of interest.

For purposes of comparison, consider the case of a wing pitching about its leading edge and translating vertically. The lift coefficient c_l and the moment coefficient about the leading edge c_m can be expressed as

$$\left.\begin{aligned} c_l &= -\frac{P}{\rho b V^2} = 4\left[-ik\left(L_1' + iL_2'\right)\alpha_h + k^2\left(L_3' + iL_4'\right)\alpha\right] \\ c_m &= \frac{M_\alpha}{2\rho b^2 V^2} = -2\left[-ik\left(M_1' + iM_2'\right)\alpha_h + k^2\left(M_3' + iM_4'\right)\alpha\right] \end{aligned}\right\} \qquad (24)$$

where $\alpha_h = \frac{\dot{h}}{V}$ is the angle of attack due to vertical translation and the quantities L_i' and M_i' are now dependent on M as well as k. For the nonoscillating wing in incompressible flow $(k = 0, M = 0)$ $c_l = 2\pi\alpha$ and $c_m = -\frac{\pi}{2}\alpha$. From equation (24) the lift- and moment-curve slopes (complex derivatives) associated with vertical translation and pitching are, respectively,

$$\left.\begin{aligned} \frac{dc_l}{d\alpha_h} &= -i4k\left(L_1' + iL_2'\right) \\ \frac{dc_m}{d\alpha_h} &= i2k\left(M_1' + iM_2'\right) \\ &\text{and} \\ \frac{dc_l}{d\alpha} &= 4k^2\left(L_3' + iL_4'\right) \\ \frac{dc_m}{d\alpha} &= 2k^2\left(M_3' + iM_4'\right) \end{aligned}\right\} \qquad (25)$$

In figure 2 the magnitudes of the slopes given by equation (25) are plotted against k for several values of M, and in figure 3 the associated phase angles are plotted. In figures 2 and 3 the dashed curves represent the supersonic results, the solid-line curves represent the subsonic results, and the solid-line curves with several of the computed points circled represent the sonic results.

In figure 2 the variation of slope with Mach number for the steady case (along ordinate $k = 0$) is given by the Prandtl-Glauert rule for subsonic speeds and the Ackeret rule for supersonic speeds. Each of these rules predicts an infinite slope at $M = 1$. In the figure, the values for the slope magnitude become excessive only for Mach numbers approaching unity and values of k approaching zero. In this neighborhood the linearized theory does not apply, and the Mach number and

k range in which the theory is applicable awaits experimental or theoretical determination. In figure 3 the phase-angle curves for $M = 1$ depart from those for the other Mach numbers in the low k range. At $k = 0$, the phase angle for $M = 1$ differs from the constant phase angle of all the other Mach numbers by 45°.

Figure 4 contains a cross plot against Mach number of figure 2(a) for several values of k. Note that the maximum lift-curve slope occurs at $M = 1$ only for small values of k. Above a k of around 0.2, as may also be noted in figure 2(a), the maximum lift-curve slope for a particular value of k occurs at a Mach number less than 1.

Some applications to bending-torsion flutter.- In reference 5 a systematic numerical study of the bending-torsion flutter of a two-dimensional wing was made including, among other considerations, the effect of Mach number on this type of flutter. The results were presented in the form of figures. Table I of the present paper is used to obtain points at $M = 1$ for figures 18 and 19 of reference 5. These figures of reference 5 with the $M = 1$ points added are presented as figures 5 and 6.

In figure 5 the flutter-speed coefficient $V/b\omega_{\alpha}$ is plotted against Mach number M for several values of the density parameter $1/\kappa$, for wings with the center of gravity at 60 percent chord and the elastic axis at 50 percent chord. The points for Mach number 1, indicated by circles, are consistent with the results of reference 5. As a matter of possible interest some values of the reduced frequency are indicated at $M = 0$ and $M = 1$.

In figure 6 a plot of the flutter-speed coefficient $V/b\omega_{\alpha}$ against the ratio of wing bending frequency to wing torsional frequency ω_h/ω_{α} is shown for several Mach numbers. The curve for $M = 1$, calculated points of which are circled, is shown in relation to the curves previously given in reference 5.

CONCLUDING REMARKS

The linearized theory for compressible unsteady flow has been used to obtain the forces and moments for a thin, harmonically oscillating, two-dimensional wing-aileron combination moving at sonic speed. These forces and moments and the flutter results obtained from them were found to be consistent with similar calculations previously obtained for other Mach numbers. In assessing or applying the results for a Mach number

of 1, the limitations associated with linearized theory should be kept in mind.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., September 4, 1951

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TABLE I.- VALUES OF FUNCTIONS FOR WING FLUTTER CALCULATIONS

k	k^2L_1'	k^2L_2'	k^2L_3'	k^2L_4'	k^2M_1'	k^2M_2'	k^2M_3'	k^2M_4'
3.5	-0.092268	3.5956	0.97979	3.4024	-0.13699	3.8153	0.92484	4.6249
3.0	-.13457	3.0365	.99531	2.8776	-.25228	3.2404	.93852	3.9240
2.5	-.14084	2.4591	1.0199	2.3562	-.31793	2.6183	.96756	3.2302
2.0	-.10268	1.8845	1.0503	1.8409	-.31343	1.9795	1.0071	2.5514
1.5	-.023461	1.3390	1.0808	1.3296	-.23499	1.3642	1.0459	1.8920
1.0	.077100	.84912	1.1048	.80532	-.10151	.81581	1.0653	1.2439
.8	.11455	.67354	1.1134	.57971	-.042407	.62412	1.0625	.97907
.6	.14412	.51052	1.1261	.32969	.013043	.45119	1.0525	.69934
.4	.15861	.35939	1.1582	.024216	.057482	.29803	1.0398	.38091
.36	.15871	.33031	1.1709	-.049992	.064095	.26975	1.0387	.30774
.32	.15754	.30149	1.1875	-.13156	.069652	.24221	1.0392	.22918
.28	.15489	.27281	1.2101	-.22295	.073971	.21535	1.0419	.14346
.24	.15050	.24413	1.2414	-.32793	.076821	.18912	1.0488	.047825
.20	.14396	.21524	1.2862	-.45272	.077892	.16338	1.0627	-.062204
.18	.13974	.20060	1.3163	-.52575	.077634	.15065	1.0738	-.12488
.16	.13475	.18577	1.3538	-.60859	.076751	.13796	1.0890	-.19460
.14	.12887	.17064	1.4015	-.70450	.075154	.12527	1.1101	-.27367
.12	.12195	.15510	1.4638	-.81857	.072717	.11249	1.1399	-.36562
.10	.11376	.13898	1.5480	-.95913	.069272	.099537	1.1830	-.47627
.09	.10907	.13061	1.6023	-1.0437	.067098	.092939	1.2120	-.54159
.08	.10394	.12198	1.6681	-1.1414	.064570	.086227	1.2482	-.61613
.07	.098250	.11302	1.7495	-1.2569	.061627	.079356	1.2941	-.70295
.06	.091915	.10364	1.8530	-1.3969	.058198	.072266	1.3541	-.80683
.05	.084783	.093703	1.9895	-1.5732	.054173	.064875	1.4352	-.93558
.045	.080848	.088464	2.0756	-1.6808	.051891	.061029	1.4873	-1.0133
.04	.076618	.083002	2.1789	-1.8066	.049394	.057053	1.5503	-1.1019
.035	.072039	.077263	2.3050	-1.9566	.046644	.052913	1.6283	-1.2103
.03	.067037	.071182	2.4632	-2.1407	.043593	.048569	1.7271	-1.3402
.025	.061508	.064663	2.6693	-2.3749	.040169	.043954	1.8573	-1.5041
.024	.060324	.063291	2.7184	-2.4300	.039431	.042991	1.8885	-1.5424
.023	.059116	.061898	2.7708	-2.4885	.038673	.042013	1.9219	-1.5830
.022	.057877	.060481	2.8269	-2.5508	.037893	.041018	1.9576	-1.6262
.021	.056602	.059032	2.8871	-2.6173	.037091	.040004	1.9961	-1.6723
.020	.055296	.057552	2.9519	-2.6886	.036264	.038972	2.0376	-1.7216
.019	.053948	.056038	3.0219	-2.7653	.035411	.037919	2.0825	-1.7746
.018	.052563	.054490	3.0978	-2.8480	.034532	.036842	2.1314	-1.8316
.017	.051133	.052901	3.1804	-2.9377	.033619	.035741	2.1847	-1.8934
.016	.049656	.051272	3.2712	-3.0356	.032676	.034614	2.2432	-1.9606
.015	.048130	.049595	3.3710	-3.1428	.031696	.033455	2.3078	-2.0341
.014	.046544	.047865	3.4814	-3.2610	.030678	.032264	2.3794	-2.1150
.013	.044897	.046080	3.6048	-3.3925	.029617	.031035	2.4595	-2.2048
.012	.043178	.044227	3.7434	-3.5395	.028506	.029765	2.5498	-2.3050
.011	.041382	.042303	3.9012	-3.7059	.027344	.028448	2.6526	-2.4183
.010	.039496	.040294	4.0823	-3.8961	.026118	.027076	2.7710	-2.5476

TABLE II.- VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS

x_1 (wing chords)	k^2L_5'	k^2L_6'	k^2N_1'	k^2N_2'	k^2N_3'	k^2N_4'	k^2N_5'	k^2N_6'
				k = 3.5				
0.1	0.89067	2.7314	-0.09339	3.1344	0.74199	3.9401	0.12031	-0.86753
.2	.80334	2.1347	-.045042	2.5061	.58009	3.2668	.60700	2.3329
.3	.71591	1.6131	-.0065088	1.9349	.43962	2.6203	.47589	1.5491
.4	.62642	1.1660	.018221	1.4282	.32026	2.0135	.35957	.96683
.5	.53285	.79307	.028928	.99321	.22116	1.4603	.25674	.55513
.6	.43448	.49071	.029296	.63394	.14121	.97433	.16833	.28195
.7	.33082	.25786	.021995	.35537	.079711	.57080	.095843	.11782
.8	.22375	.091798	.012059	.15626	.035526	.26349	.042332	.033681
.9	.11746	-.0069616	.0034955	.038640	.0089560	.068345	.010386	.0028866
				k = 3.0				
0.1	0.90819	2.3077	-0.20128	2.6676	0.75299	3.3431	0.59062	-1.0036
.2	.82052	1.8021	-.14342	2.1398	.58836	2.7714	.62394	1.9796
.3	.73081	1.3604	-.092088	1.6590	.44511	2.2224	.48942	1.3160
.4	.63775	.98181	-.051800	1.2308	.32322	1.7075	.36851	.82271
.5	.54039	.66478	-.023670	.86078	.22208	1.2382	.26148	.47300
.6	.43845	.40760	-.0068100	.55321	.14093	.82635	.16982	.24055
.7	.33269	.20837	.00094230	.31164	.078792	.48402	.095832	.10013
.8	.22522	.065938	.0025070	.13836	.034885	.22367	.042098	.027973
.9	.11980	-.017583	.0011246	.034469	.0087147	.058061	.010400	.0018736
				k = 2.5				
0.1	0.93119	1.8879	-0.26703	2.1572	0.77781	2.7522	1.0333	-0.90819
.2	.84025	1.4727	-.20750	1.7337	.60881	2.2808	.64456	1.6329
.3	.74619	1.1096	-.15129	1.3481	.46103	1.8281	.50353	1.0874
.4	.64844	.79775	-.10298	1.0040	.33475	1.4039	.37653	.68113
.5	.54688	.53640	-.064825	.70575	.22984	1.0179	.26473	.39246
.6	.44193	.32213	-.036076	.45578	.14533	.67894	.17045	.19903
.7	.33476	.15607	-.017119	.25831	.080881	.39763	.095456	.082000
.8	.22755	.037479	-.0061219	.11543	.035620	.18376	.041826	.021904
.9	.12326	-.030013	-.0011501	.028958	.0088363	.047711	.010466	.00072869
				k = 2.0				
0.1	0.95660	1.4727	-0.27189	1.6293	0.81216	2.1748	1.3413	-0.63396
.2	.86004	1.1456	-.22041	1.3098	.63808	1.8017	.66496	1.2943
.3	.76036	.85876	-.16926	1.0197	.48488	1.4432	.51552	.86344
.4	.65768	.61140	-.12269	.76104	.35314	1.1075	.38209	.54124
.5	.55240	.40268	-.082900	.53616	.24290	.80216	.26633	.31098
.6	.44536	.23188	-.050992	.34768	.15385	.53476	.17000	.15666
.7	.33782	.098908	-.027244	.19788	.085640	.31294	.094720	.062940
.8	.23164	.0048432	-.011370	.088860	.037677	.14454	.041592	.015236
.9	.12862	-.045272	-.0026384	.022415	.0093252	.037512	.010627	-.00062204
				k = 1.5				
0.1	0.97994	1.0577	-0.21153	1.1188	0.84629	1.6153	1.4453	-0.28161
.2	.87691	.81520	-.17744	.89766	.66784	1.3386	.67928	.96219
.3	.77191	.60170	-.14113	.69811	.50999	1.0719	.52182	.64145
.4	.66539	.41675	-.10616	.52088	.37404	.82193	.38333	.40055
.5	.55793	.26010	-.074653	.36716	.25792	.59472	.26516	.22779
.6	.45043	.13188	-.047957	.23837	.16412	.39602	.16839	.11189
.7	.34394	.032740	-.026874	.13591	.091712	.23148	.093816	.042001
.8	.23960	-.035165	-.011820	.061184	.040478	.10679	.041603	.0074945
.9	.13761	-.065466	-.0029068	.015481	.010045	.027684	.010987	-.0023279
				k = 1.0				
0.1	0.99808	0.62511	-0.10111	0.66383	0.86349	1.0685	1.3319	0.015814
.2	.89071	.46377	-.090344	.52958	.68378	.88808	.68000	.62662
.3	.78309	.32129	-.075513	.41001	.52446	.71223	.51857	.41259
.4	.67565	.19781	-.05929	.30482	.38573	.54649	.37888	.25176
.5	.56887	.093697	-.043368	.21425	.26795	.39542	.26141	.13690
.6	.46329	.0096862	-.028919	.13880	.17142	.26317	.16637	.060946
.7	.35939	-.052749	-.016806	.079021	.096326	.15370	.093606	.016863
.8	.25724	-.090548	-.0076650	.035540	.042741	.070828	.042507	-.0024881
.9	.15480	-.095913	-.0019552	.0089892	.010663	.018337	.011830	-.0047628
				k = 0.8				
0.1	1.0058	0.43534	-0.050987	0.50518	0.86067	0.84646	1.2356	0.083840
.2	.89824	.30614	-.049679	.40143	.68186	.70630	.67520	.48465
.3	.79104	.19227	-.043885	.30975	.52352	.56794	.51439	.31359
.4	.68454	.094067	-.03587	.22961	.38556	.43656	.37595	.18566
.5	.57912	.012108	-.027084	.16099	.26827	.31627	.25995	.095226
.6	.47503	-.052625	-.018542	.10405	.17194	.21067	.16625	.036670
.7	.37242	-.098381	-.011024	.059124	.096806	.12310	.094394	.0043043
.8	.27076	-.12172	-.0051309	.026545	.043044	.056737	.043561	-.0077843
.9	.16681	-.11414	-.0013332	.0067040	.010760	.014689	.012482	-.0061613

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TABLE II.- VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS - Continued

x_1 (wing chords)	k^2L_5'	k^2L_6'	k^2N_1'	k^2N_2'	k^2N_3'	k^2N_4'	k^2N_5'	k^2N_6'
k = 0.6								
0.1	1.0191	0.22164	-0.0032162	0.36259	0.85075	0.61351	1.1206	0.10198
.2	.91274	.12542	-.010365	.28651	.67352	.51660	.66834	.33006
.3	.80716	.041375	-.012874	.22003	.51710	.41810	.50994	.20378
.4	.70250	-.029996	-.012425	.16257	.38102	.32298	.37393	.11079
.5	.59900	-.087916	-.010642	.11344	.26534	.23487	.26002	.046840
.6	.49655	-.13117	-.0078988	.073069	.17025	.15691	.16781	.0076522
.7	.39488	-.15773	-.0049936	.041386	.095990	.091897	.096638	-.011239
.8	.29274	-.16371	-.0024409	.018527	.042746	.042430	.045598	-.014625
.9	.18530	-.13969	-.00066060	.0046667	.010705	.011000	.013541	-.0080683
k = 0.4								
0.1	1.0538	-0.044994	0.036142	0.23706	0.83622	0.35098	1.0027	0.043686
.2	.95006	-.10525	.022694	.18582	.66002	.30456	.66499	.14668
.3	.84709	-.15606	.013670	.14172	.50573	.25195	.51053	.070298
.4	.74483	-.19675	.0076867	.10395	.37218	.19792	.37757	.017218
.5	.64310	-.22637	.0038858	.072186	.25899	.14588	.26566	-.015551
.6	.54152	-.24344	.0016488	.046250	.16614	.098523	.17424	-.031136
.7	.43912	-.24557	.00049747	.026066	.093675	.058224	.10259	-.032906
.8	.33362	-.22829	.000045360	.011615	.020179	.0055347	.049926	-.024645
.9	.21789	-.18066	-.000029310	.0029130	.010457	.0070666	.015504	-.011035
k = 0.36								
0.1	1.1252	0.030351	0.042228	0.21401	0.83394	0.29112	0.84152	0.19224
.2	1.0089	-.068452	.027934	.16742	.65750	.25654	.66627	.10323
.3	.89191	-.15063	.017956	.12745	.50339	.21455	.51262	.038111
.4	.77183	-.21569	.011013	.093337	.37025	.16994	.38017	-.0057839
.5	.65815	-.26288	.0063034	.064718	.25752	.12607	.26844	-.031221
.6	.54229	-.29087	.0032579	.041408	.16512	.085600	.17685	-.041114
.7	.42738	-.29735	.0014338	.023307	.093079	.050808	.10474	-.038632
.8	.31278	-.27758	.00047434	.010373	.041458	.023727	.051359	-.027377
.9	.19441	-.21923	.000080716	.0025984	.010388	.0062091	.016112	-.011873
k = 0.32								
0.1	1.1418	-0.056244	0.047477	0.19161	0.83258	0.22710	0.84325	0.13065
.2	1.0257	-.14163	.032523	.14957	.65555	.20533	.66906	.056003
.3	.90938	-.21188	.021755	.11364	.50139	.17477	.51612	.0028801
.4	.79305	-.26507	.013986	.083076	.36845	.14025	.38609	-.033420
.5	.67689	-.30429	.0084806	.057511	.25611	.10510	.27225	-.048651
.6	.56134	-.32414	.0054099	.037434	.16215	.075794	.18028	-.052319
.7	.44617	-.32337	.0022870	.020650	.092482	.042991	.10745	-.045130
.8	.33022	-.29694	.00086655	.0091777	.041178	.020187	.053123	-.030513
.9	.20839	-.23187	.00018225	.0022962	.010314	.0053115	.016845	-.012849
k = 0.28								
0.1	1.1638	-0.15171	0.051744	0.16984	0.83284	0.15751	0.84743	0.062999
.2	1.0482	-.22324	.036348	.13225	.65471	.14985	.67411	.0037623
.3	.93233	-.28100	.024976	.10027	.50009	.13180	.52167	-.036387
.4	.81646	-.32435	.016540	.073161	.36710	.10826	.38964	-.059668
.5	.70073	-.35225	.010372	.050555	.25495	.082571	.27754	-.068418
.6	.58515	-.36313	.0059935	.032242	.16328	.057313	.18483	-.065151
.7	.46924	-.35420	.0030403	.018093	.091940	.034632	.11094	-.052654
.8	.35129	-.32008	.0012156	.0080289	.040915	.016414	.055339	-.034189
.9	.22504	-.24707	.00027249	.0020056	.010245	.0043502	.017746	-.014009
k = 0.24								
0.1	1.1940	-0.25965	0.054843	0.14865	0.83601	0.080237	0.85536	-0.013014
.2	1.0788	-.31485	.039262	.11545	.65583	.088468	.68636	-.059414
.3	.96319	-.36111	.027507	.087322	.50013	.084407	.53034	-.081216
.4	.84758	-.39238	.018594	.0635[illegible]3	.36662	.073094	.39792	-.092506
.5	.73187	-.40928	.011918	.043844	.25432	.057876	.28498	-.091405
.6	.61574	-.41006	.0070537	.027810	.16270	.041317	.19102	-.080231
.7	.49839	-.39174	.0036728	.015634	.091538	.025521	.11556	-.061597
.8	.37749	-.34853	.0024368	.0078515	.037167	.019152	.058193	-.038616
.9	.24542	-.26587	.00034985	.0017273	.010186	.0033103	.018885	-.015424
k = 0.20								
0.1	1.1847	-0.47320	0.056532	0.12795	0.84412	-0.0082108	0.93132	-0.17059
.2	1.0830	-.48687	.041076	.099080	.66048	.018512	.69696	-.12455
.3	.98104	-.49316	.029205	.074748	.50264	.030608	.54396	-.13410
.4	.87824	-.49112	.020040	.054292	.36780	.033313	.41036	-.13162
.5	.77400	-.47956	.013047	.037358	.25474	.030037	.29575	-.11907
.6	.66724	-.45656	.0078484	.023731	.16275	.023342	.19971	-.098580
.7	.55588	-.41908	.0041572	.013266	.091452	.015313	.12187	-.072616
.8	.43576	-.36130	.0017422	.0058664	.040624	.0077280	.062012	-.044140
.9	.29519	-.26886	.00041096	.0014604	.010154	.0021518	.020376	-.017216

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TABLE II.- VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS - Continued

x_1 (wing chords)	k^2L_5'	k^2L_6'	k^2N_1'	k^2N_2'	k^2N_3'	k^2N_4'	k^2N_5'	k^2N_6'
k = 0.18								
0.1	1.2147	-0.53982	0.056745	0.11774	0.85115	-0.058372	0.93364	-0.21066
.2	1.1127	-.54727	.041491	.091031	.66501	-.021025	.70742	-.16446
.3	1.0102	-.54743	.029680	.068581	.50541	.00029990	.55362	-.16484
.4	.90658	-.53940	.020491	.049747	.36942	.010968	.41893	-.15453
.5	.80115	-.52183	.013424	.034188	.25563	.014442	.30300	-.13540
.6	.69268	-.49284	.0081269	.021694	.16318	.013299	.20544	-.10951
.7	.57899	-.44919	.0043332	.012115	.091621	.0096244	.12596	-.079234
.8	.45551	-.38485	.0018283	.0053512	.040672	.0051801	.064450	-.047492
.9	.30978	-.28480	.00043445	.0013308	.010160	.0015101	.021314	-.018316
k = 0.16								
0.1	1.2518	-0.61565	0.056466	0.10760	0.86126	-0.11399	0.94001	-0.25733
.2	1.1494	-.61627	.041518	.083052	.67174	-.064748	.72110	-.20927
.3	1.0460	-.60969	.029865	.062474	.50980	-.033137	.56604	-.19952
.4	.94113	-.59494	.020730	.045256	.37217	-.013629	.42980	-.18052
.5	.83405	-.57070	.013654	.031063	.25723	-.0026913	.31204	-.15403
.6	.72335	-.53494	.0083118	.019686	.16404	.0022897	.21250	-.12205
.7	.60667	-.48430	.0044564	.010981	.092024	.0033999	.13094	-.086886
.8	.47905	-.41244	.0018911	.0048451	.040814	.0023970	.067382	-.051395
.9	.32712	-.30356	.00045197	.0012036	.010189	.00081027	.022432	-.019606
k = 0.14								
0.1	1.2988	-0.70384	0.055619	0.097485	0.87569	-0.17684	0.95199	-0.31252
.2	1.1954	-.69678	.041107	.075111	.68167	-.11401	.73931	-.26060
.3	1.0907	-.68261	.029714	.056413	.51650	-.070717	.58228	-.23947
.4	.98410	-.66030	.020725	.040805	.37652	-.041207	.44376	-.21062
.5	.87475	-.62843	.013716	.027971	.25990	-.021854	.32354	-.17574
.6	.76111	-.58488	.0083900	.017703	.16555	-.010000	.22136	-.13676
.7	.64057	-.52610	.0045205	.0098625	.092767	.0035341	.13712	-.095915
.8	.50774	-.44543	.0019277	.0043467	.041105	-.00069611	.070985	-.056032
.9	.34814	-.32610	.00046305	.0010787	.010252	.000033518	.023794	-.021150
k = 0.12								
0.1	1.3599	-0.80908	0.054111	0.087342	0.89653	-0.24965	0.97203	-0.37930
.2	1.2548	-.79327	.040180	.067170	.69636	-.17090	.76408	-.32092
.3	1.1481	-.77034	.029172	.050365	.48341	-.11399	.60402	-.28668
.4	1.0390	-.73925	.020435	.036374	.38324	-.072883	.46224	-.24638
.5	.92650	-.69846	.013583	.024898	.26414	-.043813	.33853	-.20170
.6	.80889	-.64572	.0083434	.015736	.16802	-.024051	.23279	-.15447
.7	.68325	-.57728	.0045147	.0087552	.094032	-.011444	.14501	-.10686
.8	.54369	-.48600	.0019336	.0038536	.041617	-.0042175	.075541	-.061694
.9	.37434	-.35395	.00046633	.00095528	.010364	-.00085127	.025498	-.023050
k = 0.10								
0.1	1.4421	-0.93930	0.051808	0.077093	0.92732	-0.33689	1.0042	-0.46260
.2	1.3345	-.91314	.038635	.059172	.71843	-.23884	.79882	-.39432
.3	1.2246	-.87981	.028163	.044289	.54212	-.16551	.63413	-.34445
.4	1.1118	-.83816	.019806	.031934	.39376	-.11048	.48748	-.29046
.5	.99473	-.78657	.013214	.021824	.27091	-.069811	.35879	-.23390
.6	.87155	-.72260	.0081483	.013774	.17204	-.040637	.24805	-.17656
.7	.73896	-.64222	.0044256	.0076525	.096139	-.020758	.15544	-.12062
.8	.59038	-.53772	.0019023	.0033639	.042488	-.0083561	.081503	-.068866
.9	.40823	-.38961	.00046075	.00083249	.010573	-.0018817	.027709	-.025476
k = 0.09								
0.1	1.4948	-1.0178	0.050302	0.071892	0.94835	-0.38822	1.0271	-0.51299
.2	1.3853	-.98569	.037586	.055123	.73365	-.27870	.82175	-.43802
.3	1.2734	-.94624	.027450	.041220	.55291	-.19566	.65382	-.37902
.4	1.1580	-.89837	.019339	.029696	.40116	-.13245	.50385	-.31694
.5	1.0379	-.84038	.012926	.020278	.27572	-.084961	.37182	-.25334
.6	.91101	-.76970	.0079844	.012788	.17494	-.050287	.25780	-.18997
.7	.77391	-.68214	.0043442	.0070997	.097670	-.026166	.16205	-.12901
.8	.61956	-.56961	.0018706	.0031187	.043128	-.010754	.085253	-.073265
.9	.42933	-.41167	.00045375	.00077151	.010721	-.0024828	.029092	-.026971
k = 0.08								
0.1	1.5585	-1.1089	0.048516	0.066611	0.97466	-0.44665	1.0563	-0.57139
.2	1.4467	-1.0699	.036323	.051020	.75290	-.32399	.84998	-.48821
.3	1.3320	-1.0236	.026575	.038115	.56666	-.22987	.67789	-.41882
.4	1.2133	-.96858	.018755	.027435	.41064	-.15731	.52376	-.34755
.5	1.0894	-.90323	.012557	.018718	.28193	-.10209	.38758	-.27588
.6	.95808	-.82490	.0077690	.011795	.17869	-.061178	.26952	-.20558
.7	.81555	-.72902	.0042339	.0065434	.099667	-.032263	.16996	-.13882
.8	.65421	-.60713	.0018264	.0028721	.043974	-.013450	.089728	-.078426
.9	.45434	-.43769	.00044387	.00070995	.010923	-.0031549	.030732	-.028735

TABLE II.- VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS - Continued

x_1 (wing chords)	k^2L_5'	k^2L_6'	k^2N_1'	k^2N_2'	k^2N_3'	k^2N_4'	k^2N_5'	k^2N_6'
				k = 0.07				
0.1	1.6372	-1.2166	0.046407	0.061221	1.0085	-0.51450	1.0941	-0.64043
.2	1.5222	-1.1697	.034808	.046838	.77768	-.37651	.88543	-.54704
.3	1.4039	-1.1154	.025510	.034957	.58447	-.26945	.70800	-.46564
.4	1.2812	-1.0522	.018033	.025138	.42300	-.18604	.54851	-.38366
.5	1.1525	-.97833	.012093	.017137	.29005	-.12185	.40706	-.30257
.6	1.0155	-.89087	.0074936	.010789	.18363	-.073725	.28394	-.22413
.7	.86612	-.78518	.0040903	.0059805	.10232	-.039270	.17964	-.15051
.8	.69629	-.65224	.0017668	.0026232	.045097	-.016551	.095173	-.084599
.9	.48460	-.46902	.00043011	.00064778	.011194	-.0039242	.032722	-.030851
				k = 0.06				
0.1	1.7369	-1.3476	0.043916	0.055674	1.0528	-0.59555	1.1443	-0.72410
.2	1.6178	-1.2914	.032999	.042548	.81043	-.43902	.93114	-.61787
.3	1.4947	-1.2276	.024225	.031722	.60811	-.31647	.74657	-.52218
.4	1.3665	-1.1546	.017151	.022791	.43945	-.22011	.58003	-.42743
.5	1.2316	-1.0704	.011519	.015522	.30095	-.14523	.43178	-.33504
.6	1.0874	-.97200	.0071485	.0097646	.19029	-.088553	.30216	-.24678
.7	.92934	-.85442	.0039074	.0054083	.10591	-.047545	.19182	-.16484
.8	.74869	-.70790	.0016905	.0023700	.046631	-.020200	.10199	-.092200
.9	.52222	-.50782	.00041195	.00058482	.011560	-.0048319	.035201	-.033467
				k = 0.05				
0.1	1.8681	-1.5127	0.040965	0.049908	1.1131	-0.69565	1.2128	-0.82913
.2	1.7431	-1.4452	.030833	.038098	.85515	-.51610	.99223	-.70625
.3	1.6135	-1.3697	.022671	.028375	.64058	-.37435	.79785	-.59303
.4	1.4779	-1.2845	.016076	.020365	.46220	-.26195	.62178	-.48245
.5	1.3347	-1.1875	.010812	.013857	.31605	-.17391	.46433	-.37600
.6	1.1808	-1.0754	.0067195	.0087093	.19957	-.10670	.32603	-.27545
.7	1.0113	-.94288	.0036780	.0048198	.11093	-.057655	.20770	-.18307
.8	.81660	-.77923	.0015940	.0021103	.048778	-.024646	.11084	-.10190
.9	.57076	-.55760	.00038898	.00052030	.012077	-.0059388	.038400	-.036823
				k = 0.045				
0.1	1.9509	-1.6137	0.039279	0.046915	1.1519	-0.75597	1.2571	-0.89311
.2	1.8220	-1.5394	.029589	.035790	.88412	-.56248	1.0312	-.75988
.3	1.6881	-1.4568	.021773	.026641	.66167	-.40911	.83043	-.63613
.4	1.5478	-1.3643	.015450	.019112	.48308	-.28704	.64818	-.51605
.5	1.3992	-1.2596	.010398	.012999	.32592	-.19108	.48487	-.40107
.6	1.2391	-1.1392	.0064672	.0081656	.20566	-.11754	.34101	-.29306
.7	1.0624	-.99749	.0035423	.0045168	.11423	-.063684	.21765	-.19428
.8	.85866	-.82334	.0015355	.0019771	.050194	-.027313	.11636	-.10789
.9	.60094	-.58847	.00037489	.00048738	.012420	-.0065977	.040395	-.038898
				k = 0.04				
0.1	2.0498	-1.7318	0.037426	0.043824	1.1991	-0.82584	1.3109	-0.96766
.2	1.9162	-1.6498	.028214	.033413	.91933	-.61611	1.0781	-.82232
.3	1.7771	-1.5590	.020776	.024858	.68736	-.44926	.86957	-.68640
.4	1.6310	-1.4580	.014754	.017824	.49510	-.31602	.67986	-.55525
.5	1.4759	-1.3443	.0099366	.012116	.33802	-.21088	.50939	-.43040
.6	1.3085	-1.2143	.0061837	.0076080	.21310	-.13005	.35891	-.31370
.7	1.1230	-1.0619	.0033896	.0042058	.11829	-.070624	.22949	-.20746
.8	.90867	-.87538	.0014699	.0018402	.051938	-.030368	.12293	-.11493
.9	.63667	-.62490	.00035870	.00045342	.012837	-.0073584	.042760	-.041344
				k = 0.035				
0.1	2.1705	-1.8730	0.035377	0.040614	1.2576	-0.90842	1.3775	-1.0564
.2	2.0309	-1.7818	.026693	.030946	.96307	-.67943	1.1358	-.89653
.3	1.8854	-1.6814	.019670	.023010	.71933	-.49662	.91759	-.74623
.4	1.7323	-1.5703	.013978	.016490	.51765	-.35013	.71859	-.60203
.5	1.5691	-1.4460	.0094210	.011203	.35309	-.23417	.53936	-.46546
.6	1.3926	-1.3045	.0058673	.0070313	.22245	-.14473	.38069	-.33841
.7	1.1965	-1.1393	.0032180	.0038855	.12336	-.078792	.24387	-.22326
.8	.96920	-.93804	.0013972	.0016986	.054136	-.033945	.13089	-.12341
.9	.67984	-.66883	.00034181	.00041808	.013371	-.0082444	.045619	-.044291
				k = 0.03				
0.1	2.3217	-2.0464	0.033095	0.037250	1.3320	-1.0087	1.4622	-1.1650
.2	2.1748	-1.9440	.024989	.028365	1.0188	-.75625	1.2086	-.98712
.3	2.0210	-1.8321	.018428	.021080	.76018	-.55399	.97812	-.81945
.4	1.8587	-1.7088	.013105	.015098	.54651	-.39141	.76729	-.65939
.5	1.6854	-1.5714	.0088380	.010253	.37243	-.26235	.57694	-.50852
.6	1.4974	-1.4158	.0055076	.0064317	.23442	-.16248	.40796	-.36881
.7	1.2880	-1.2350	.0030229	.0035523	.12991	-.088626	.26183	-.24274
.8	1.0445	-1.0157	.0013127	.0015530	.056942	-.038279	.14080	-.13388
.9	.73346	-.72327	.00032172	.00038174	.014072	-.0092997	.049169	-.047946

NACA

TABLE II.- VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS - Continued

x_1 (wing chords)	k^2L_5'	k^2L_6'	k^2N_1'	k^2N_2'	k^2N_3'	k^2N_4'	k^2N_5'	k^2N_6'
				k = 0.025				
0.1	2.5186	-2.2672	0.030530	0.033691	1.4302	-1.1351	1.5738	-1.3024
.2	2.3615	-2.1509	.023067	.025634	1.0926	-.85288	1.3041	-1.1018
.3	2.1968	-2.0244	.017023	.019039	.74675	-.62606	1.0571	-.91231
.4	2.0226	-1.8858	.012113	.013629	.58480	-.44322	.83081	-.73225
.5	1.8360	-1.7319	.0081744	.0092506	.39814	-.29765	.62581	-.56335
.6	1.6329	-1.5584	.0050971	.0058001	.25036	-.18471	.44336	-.40761
.7	1.4061	-1.3578	.0027995	.0032017	.13863	-.10094	.28509	-.26766
.8	1.1415	-1.1152	.0012168	.0013987	.060719	-.043673	.15361	-.14728
.9	.80256	-.79325	.00029798	.00034401	.014969	-.010652	.053757	-.052627
				k = 0.024				
0.1	2.5654	-2.3191	0.029968	0.032942	1.4538	-1.1646	1.6006	-1.3346
.2	2.4060	-2.1996	.022650	.025065	1.1103	-.87540	1.3268	-1.1288
.3	2.2386	-2.0697	.016717	.018614	.81026	-.65969	1.0760	-.93410
.4	2.0614	-1.9274	.011897	.013323	.59403	-.45529	.84591	-.74943
.5	1.8717	-1.7698	.0080300	.0090426	.40435	-.30586	.63746	-.57629
.6	1.6650	-1.5921	.0050076	.0056691	.25422	-.18987	.45176	-.41676
.7	1.4341	-1.3867	.0027499	.0031298	.14072	-.10382	.29060	-.27355
.8	1.1646	-1.1388	.0011959	.0013666	.061661	-.044896	.15664	-.15046
.9	.81890	-.80980	.00029266	.00033636	.015205	-.010948	.054835	-.053743
				k = 0.023				
0.1	2.6154	-2.3744	0.029397	0.032188	1.4789	-1.1959	1.6292	-1.3689
.2	2.4533	-2.2513	.022222	.024488	1.1293	-.89930	1.3512	-1.1573
.3	2.2832	-2.1179	.016403	.018183	.84122	-.66067	1.0961	-.95728
.4	2.1030	-1.9717	.011675	.013014	.60391	-.46808	.86206	-.76758
.5	1.9098	-1.8100	.0078810	.0088317	.41099	-.31458	.64988	-.58999
.6	1.6993	-1.6279	.0049150	.0055365	.25834	-.19535	.46075	-.42646
.7	1.4640	-1.4176	.0026999	.0030559	.14299	-.10683	.29650	-.27979
.8	1.1890	-1.1638	.0011738	.0013347	.062634	-.046232	.15988	-.15382
.9	.83630	-.82741	.00028746	.00032830	.015457	-.011263	.055995	-.054921
				k = 0.022				
0.1	2.6689	-2.4331	0.028810	0.031420	1.5060	-1.2292	1.6599	-1.4052
.2	2.5040	-2.3065	.021781	.023901	1.1496	-.92473	1.3774	-1.1877
.3	2.3308	-2.1691	.016080	.017745	.85620	-.67963	1.1177	-.98189
.4	2.1474	-2.0190	.011447	.012699	.61449	-.48169	.87938	-.78694
.5	1.9506	-1.8529	.0077275	.0086171	.41810	-.32384	.66318	-.60456
.6	1.7360	-1.6661	.0048202	.0054014	.26275	-.20118	.47035	-.43681
.7	1.4959	-1.4505	.0026477	.0029813	.14539	-.11008	.30280	-.28645
.8	1.2152	-1.1905	.0011512	.0013020	.063661	-.047664	.16335	-.15741
.9	.85489	-.84618	.00028163	.00032059	.015677	-.011643	.057233	-.056173
				k = 0.021				
0.1	2.7263	-2.4959	0.028205	0.030639	1.5352	-1.2645	1.6929	-1.4441
.2	2.5584	-2.3653	.021326	.023305	1.1715	-.95181	1.4054	-1.2201
.3	2.3820	-2.2240	.015747	.017300	.87230	-.69978	1.1409	-1.0082
.4	2.1949	-2.0695	.011211	.012379	.62591	-.49617	.89792	-.80765
.5	1.9942	-1.8988	.0075693	.0083984	.42579	-.33369	.67742	-.62018
.6	1.7752	-1.7069	.0047218	.0052642	.26754	-.20737	.48065	.44788
.7	1.5300	-1.4856	.0025940	.0029055	.14801	-.11350	.30955	-.29356
.8	1.2432	-1.2191	.0011283	.0012685	.064818	-.049141	.16705	-.16125
.9	.87481	-.86630	.00027604	.00031230	.015974	-.011994	.058556	-.057515
				k = 0.020				
0.1	2.7880	-2.5632	0.027582	0.029843	1.5665	-1.3025	1.7284	-1.4857
.2	2.6169	-2.4285	.020859	.022696	1.1951	-.98076	1.4357	-1.2547
.3	2.4370	-2.2828	.015404	.016846	.88968	-.72136	1.1659	-1.0363
.4	2.2461	-2.1237	.010968	.012053	.63824	-.51164	.91792	-.82980
.5	2.0411	-1.9480	.0074064	.0081768	.43408	-.34422	.69276	-.63688
.6	1.8174	-1.7508	.0046212	.0051240	.27269	-.21399	.49172	-.45972
.7	1.5667	-1.5234	.0025391	.0028277	.15082	-.11717	.31681	-.30120
.8	1.2733	-1.2498	.0011040	.0012350	.065984	-.050796	.17104	-.16537
.9	.89624	-.88788	.00027078	.00030336	.016262	-.012401	.059980	-.058960
				k = 0.019				
0.1	2.8548	-2.6357	0.026938	0.029032	1.6005	-1.3432	1.7670	-1.5304
.2	2.6801	-2.4965	.020375	.022076	1.2208	-1.0118	1.4684	-1.2921
.3	2.4964	-2.3461	.015049	.016384	.90853	-.74445	1.1929	-1.0666
.4	2.3013	-2.1820	.010717	.011721	.65164	-.52822	.93954	-.85362
.5	2.0918	-2.0011	.0072381	.0079503	.44309	-.35549	.70933	-.65485
.6	1.8629	-1.7980	.0045165	.0049818	.27829	-.22108	.50367	-.47251
.7	1.6063	-1.5641	.0024821	.0027487	.15390	-.12108	.32464	-.30943
.8	1.3058	-1.2828	.0010796	.0012001	.067334	-.052500	.17533	-.16981
.9	.91929	-.91116	.00026495	.00029461	.016595	-.012818	.061514	-.060518

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TABLE II.- VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS - Continued

x_1 (wing chords)	k^2L_5'	k^2L_6'	k^2N_1'	k^2N_2'	k^2N_3'	k^2N_4'	k^2N_5'	k^2N_6'
				k = 0.018				
0.1	2.9271	-2.7139	0.026272	0.028204	1.6375	-1.3871	1.8089	-1.5786
.2	2.7486	-2.5699	.019874	.021444	1.2486	-1.0452	1.5038	-1.3323
.3	2.5607	-2.4144	.014681	.015914	.92904	-.76934	1.2209	-1.0993
.4	2.3612	-2.2451	.010456	.011383	.66621	-.54607	.96296	-.87937
.5	2.1467	-2.0583	.0070622	.0077206	.45289	-.36764	.72728	-.67428
.6	1.9122	-1.8490	.0044074	.0048376	.28440	-.22870	.51665	-.48629
.7	1.6492	-1.6081	.0024221	.0026691	.15725	-.12529	.33310	-.31833
.8	1.3410	-1.3186	.0010533	.0011654	.068795	-.054332	.17999	-.17462
.9	.94427	-.93636	.00025855	.00028609	.016983	-.013238	.063170	-.062205
				k = 0.017				
0.1	3.0059	-2.7987	0.025583	0.027357	1.6779	-1.4345	1.8546	-1.6308
.2	2.8232	-2.6496	.019356	.020797	1.2791	-1.0814	1.5426	-1.3759
.3	2.6308	-2.4887	.014300	.015431	.95150	-.79625	1.2541	-1.1347
.4	2.4263	-2.3135	.010187	.011037	.68216	-.56537	.98850	-.90732
.5	2.2064	-2.1206	.0068811	.0074851	.46364	-.38073	.74683	-.69539
.6	1.9658	-1.9044	.0042948	.0046893	.29111	-.23691	.53072	-.50130
.7	1.6958	-1.6559	.0023608	.0025869	.16094	-.12982	.34232	-.32802
.8	1.3792	-1.3575	.0010270	.0011292	.070418	-.056294	.18504	-.17985
.9	.97142	-.96373	.00025201	.00027727	.017388	-.013714	.064967	-.064042
				k = 0.016				
0.1	3.0922	-2.8913	0.024870	0.026488	1.7222	-1.4861	1.9048	-1.6877
.2	2.9048	-2.7364	.018820	.020134	1.3125	-1.1208	1.5850	-1.4233
.3	2.7075	-2.5695	.013906	.014938	.97610	-.82552	1.2891	-1.1733
.4	2.4976	-2.3881	.0099067	.010683	.69962	-.58637	1.0165	-.93768
.5	2.2717	-2.1884	.0066931	.0072438	.47542	-.39501	.76831	-.71834
.6	2.0244	-1.9649	.0041779	.0045379	.29842	-.24589	.54620	-.51761
.7	1.7468	-1.7081	.0022970	.0025028	.16495	-.13479	.35246	-.33851
.8	1.4210	-1.3999	.00099935	.0010925	.072133	-.058488	.19059	-.18553
.9	1.0011	-.99364	.00024528	.00026819	.017780	-.014281	.066939	-.066043
				k = 0.015				
0.1	3.1871	-2.9925	0.024129	0.025598	1.7712	-1.5426	1.9601	-1.7499
.2	2.9948	-2.8316	.018262	.019455	1.3494	-1.1638	1.6319	-1.4752
.3	2.7918	-2.6584	.013495	.014432	1.0033	-.85752	1.3277	-1.2155
.4	2.5760	-2.4701	.0096154	.010320	.71894	-.60928	1.0473	-.97099
.5	2.3436	-2.2628	.0064969	.0069973	.48843	-.41063	.79191	-.74351
.6	2.0889	-2.0312	.0040561	.0043828	.30652	-.25569	.56320	-.53548
.7	1.8028	-1.7654	.0022300	.0024172	.16937	-.14022	.36358	-.35006
.8	1.4669	-1.4465	.00097034	.0010550	.074050	-.060865	.19668	-.19177
.9	1.0337	-1.0265	.00023810	.00025902	.018238	-.014878	.069109	-.068234
				k = 0.014				
0.1	3.2924	-3.1042	0.023357	0.024682	1.8255	-1.6047	2.0215	-1.8185
.2	3.0943	-2.9367	.017680	.018757	1.3905	-1.2111	1.6838	-1.5324
.3	2.8853	-2.7563	.013068	.013912	1.0335	-.89270	1.3705	-1.2621
.4	2.6629	-2.5603	.0093118	.0099470	.74045	-.63449	1.0814	-1.0077
.5	2.4230	-2.3451	.0062926	.0067438	.50292	-.42775	.81805	-.77128
.6	2.1603	-2.1045	.0039288	.0042238	.31554	-.26642	.58200	-.55525
.7	1.8648	-1.8286	.0021603	.0023293	.17433	-.14615	.37587	-.36282
.8	1.5177	-1.4980	.00094017	.0010165	.076211	-.063451	.20341	-.19869
.9	1.0697	-1.0627	.00023065	.00024963	.018772	-.015509	.071516	-.070646
				k = 0.013				
0.1	3.4097	-3.2286	0.022555	0.023739	1.8864	-1.6735	2.0902	-1.8945
.2	3.2053	-3.0533	.017074	.018037	1.4364	-1.2635	1.7417	-1.5960
.3	2.9894	-2.8651	.012621	.013378	1.0674	-.93165	1.4181	-1.3138
.4	2.7594	-2.6607	.0089943	.0095637	.76454	-.66238	1.1195	-1.0486
.5	2.5115	-2.4365	.0060789	.0064832	.51920	-.44665	.84715	-.80219
.6	2.2398	-2.1860	.0037957	.0040601	.32570	-.27828	.60292	-.57725
.7	1.9339	-1.8991	.0020873	.0022387	.17993	-.15267	.38955	-.37704
.8	1.5743	-1.5553	.00090843	.00097697	.078678	-.066270	.21090	-.20638
.9	1.1098	-1.1031	.00022294	.00023984	.019430	-.016150	.074147	-.073381
				k = 0.012				
0.1	3.5417	-3.3676	0.021712	0.022764	1.9549	-1.7505	2.1675	-1.9797
.2	3.3301	-3.1841	.016439	.017294	1.4882	-1.3221	1.8071	-1.6669
.3	3.1065	-2.9871	.012153	.012825	1.1056	-.97518	1.4720	-1.3716
.4	2.8682	-2.7734	.0086626	.0091670	.79170	-.69356	1.1624	-1.0942
.5	2.6112	-2.5390	.0058552	.0062137	.53749	-.46784	.87996	-.83673
.6	2.3291	-2.2775	.0036566	.0038909	.33710	-.29157	.62656	-.60182
.7	2.0114	-1.9780	.0020110	.0021455	.18618	-.16003	.40496	-.39290
.8	1.6379	-1.6196	.00087519	.00093623	.081361	-.069513	.21933	-.21498
.9	1.1549	-1.1484	.00021465	.00023000	.020058	-.016976	.077175	-.076378

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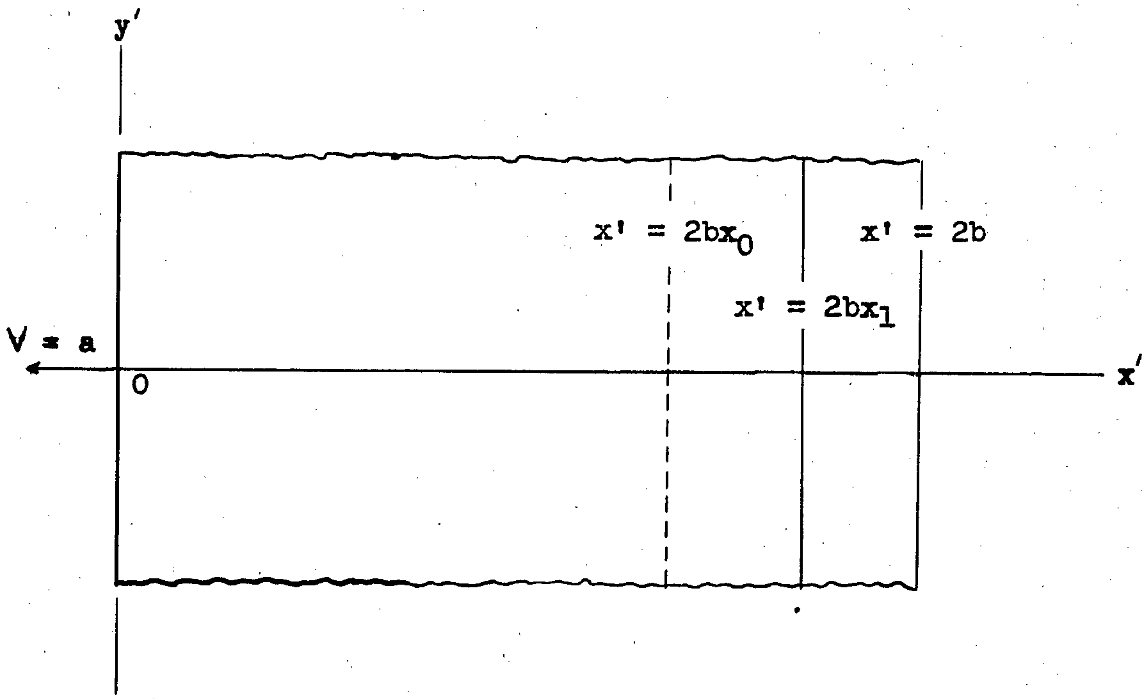
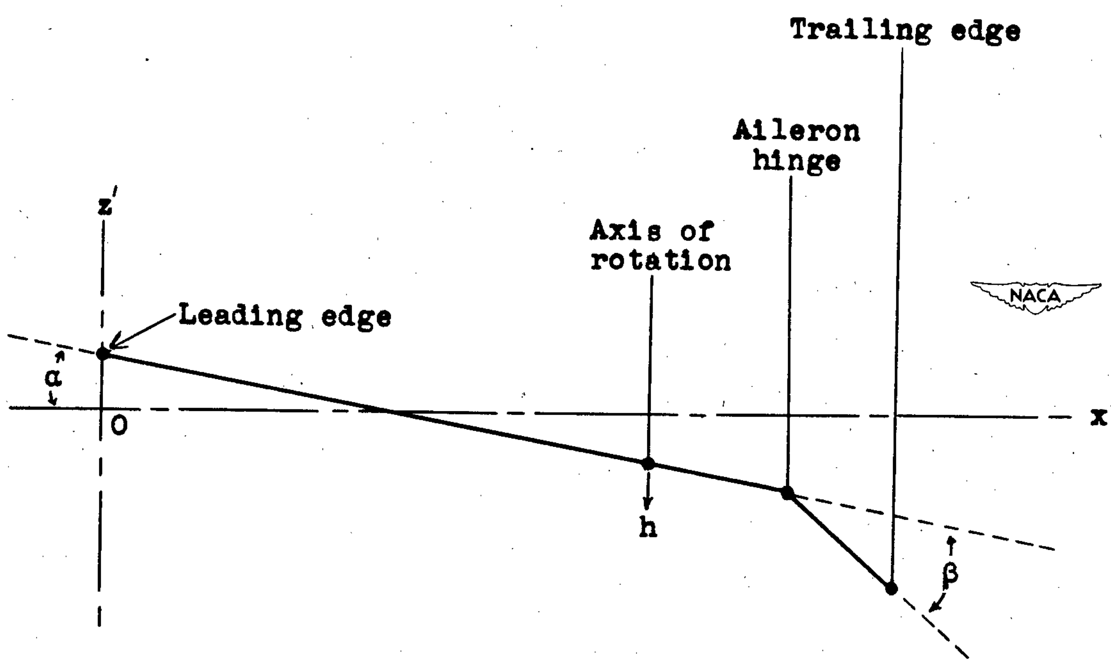
TABLE II.- VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS - Concluded

x_1 (wing chords)	k^2L_5'	k^2L_6'	k^2N_1'	k^2N_2'	k^2N_3'	k^2N_4'	k^2N_5'	k^2N_6'
				k = 0.011				
0.1	3.6917	-3.5250	0.020830	0.021752	2.0329	-1.8373	2.2558	-2.0756
.2	3.4719	-3.3322	.015774	.016523	1.5471	-1.3881	1.8814	-1.7472
.3	3.2395	-3.1252	.011663	.012251	1.1491	-1.0243	1.5331	-1.4370
.4	2.9917	-2.9010	.0083139	.0087565	.82262	-.72876	1.2112	-1.1458
.5	2.7242	-2.6551	.0056202	.0059347	.55834	-.49176	.91728	-.87576
.6	2.4304	-2.3810	.0035105	.0037155	.35009	-.30658	.65338	-.62964
.7	2.0995	-2.0674	.0019308	.0020485	.19327	-.16835	.42248	-.41087
.8	1.7099	-1.6923	.00084042	.00089384	.084414	-.077263	.23988	-.23588
.9	1.2059	-1.1998	.00020593	.00021976	.020733	-.017948	.080575	-.079806
				k = 0.010				
0.1	3.8640	-3.7050	0.019900	0.020700	2.1229	-1.9363	2.3572	-2.1855
.2	3.6347	-3.5015	.015072	.015722	1.6152	-1.4636	1.9668	-1.8390
.3	3.3922	-3.2832	.011145	.011656	1.1993	-1.0802	1.6034	-1.5117
.4	3.1334	-3.0469	.0079462	.0083296	.85836	-.76880	1.2672	-1.2048
.5	2.8538	-2.7880	.0053724	.0056448	.58247	-.51893	.96003	-.92053
.6	2.5467	-2.4996	.0033557	.0035340	.36512	-.32364	.68415	-.66149
.7	2.2004	-2.1698	.0018459	.0019482	.20154	-.17775	.44253	-.43150
.8	1.7924	-1.7758	.00080360	.00084993	.088031	-.077263	.23988	-.23588
.9	1.2645	-1.2586	.00019691	.00020898	.021638	-.018934	.084451	-.083757

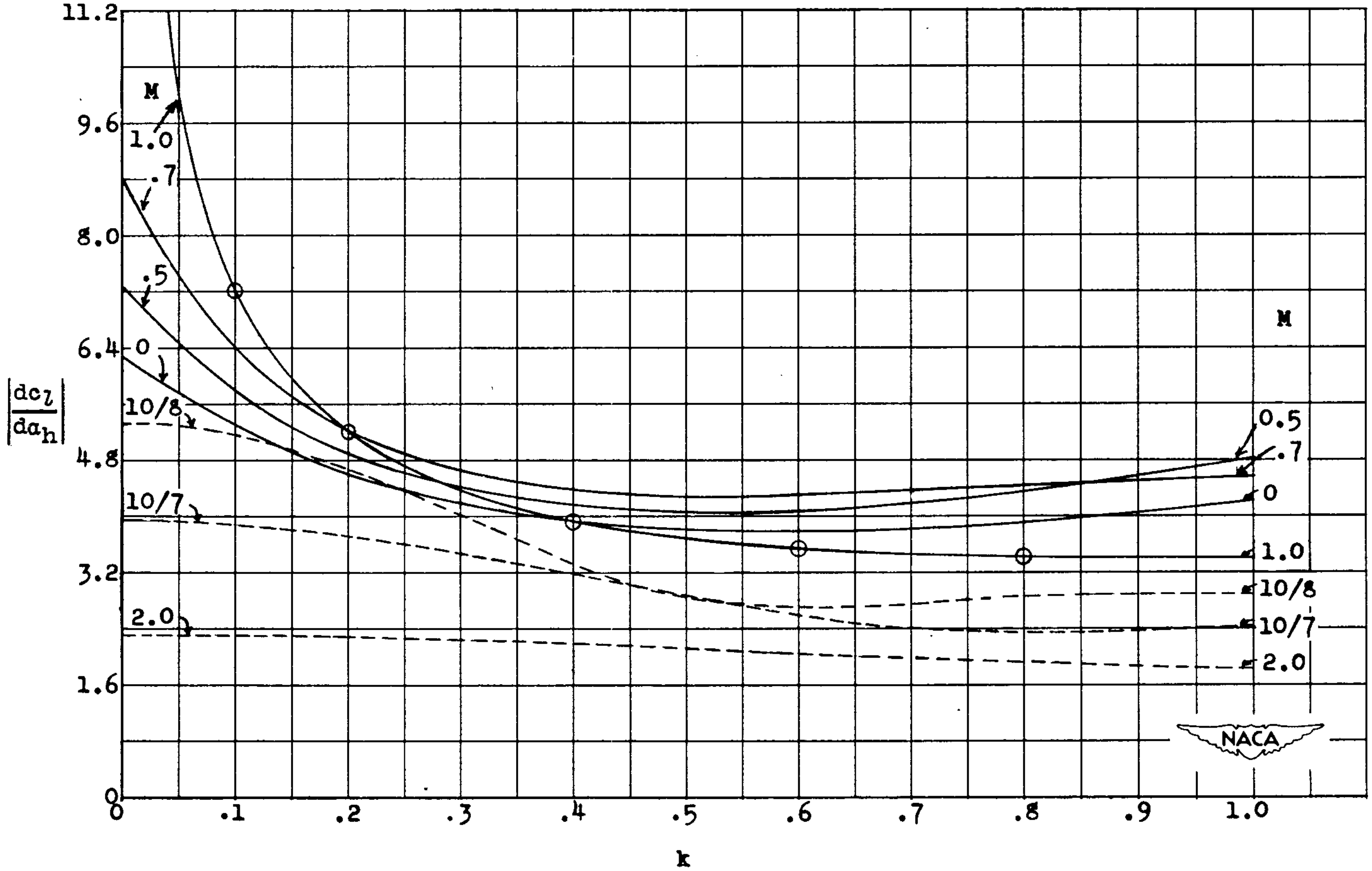


(a) Projection of wing strip on x'y'-plane.



(b) Section $y' = 0$.

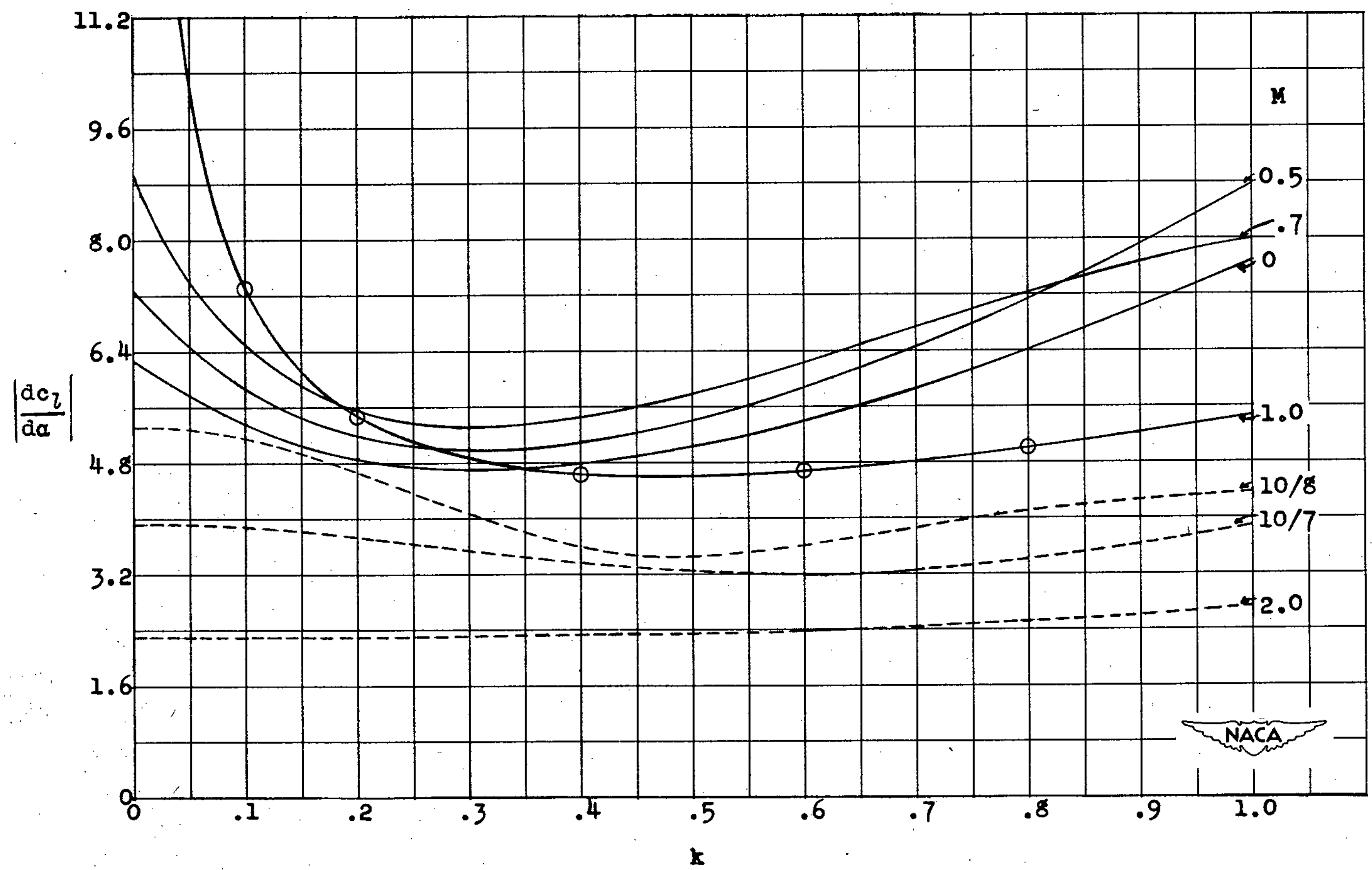
Figure 1.- Sketch illustrating coordinate system and the degrees of freedom α, h, and β.



(a) Lift-curve slope associated with vertical translation of wing.

$$\left|\frac{dc_l}{d\alpha_h}\right| = 4k\sqrt{L_1'^2 + L_2'^2}.$$

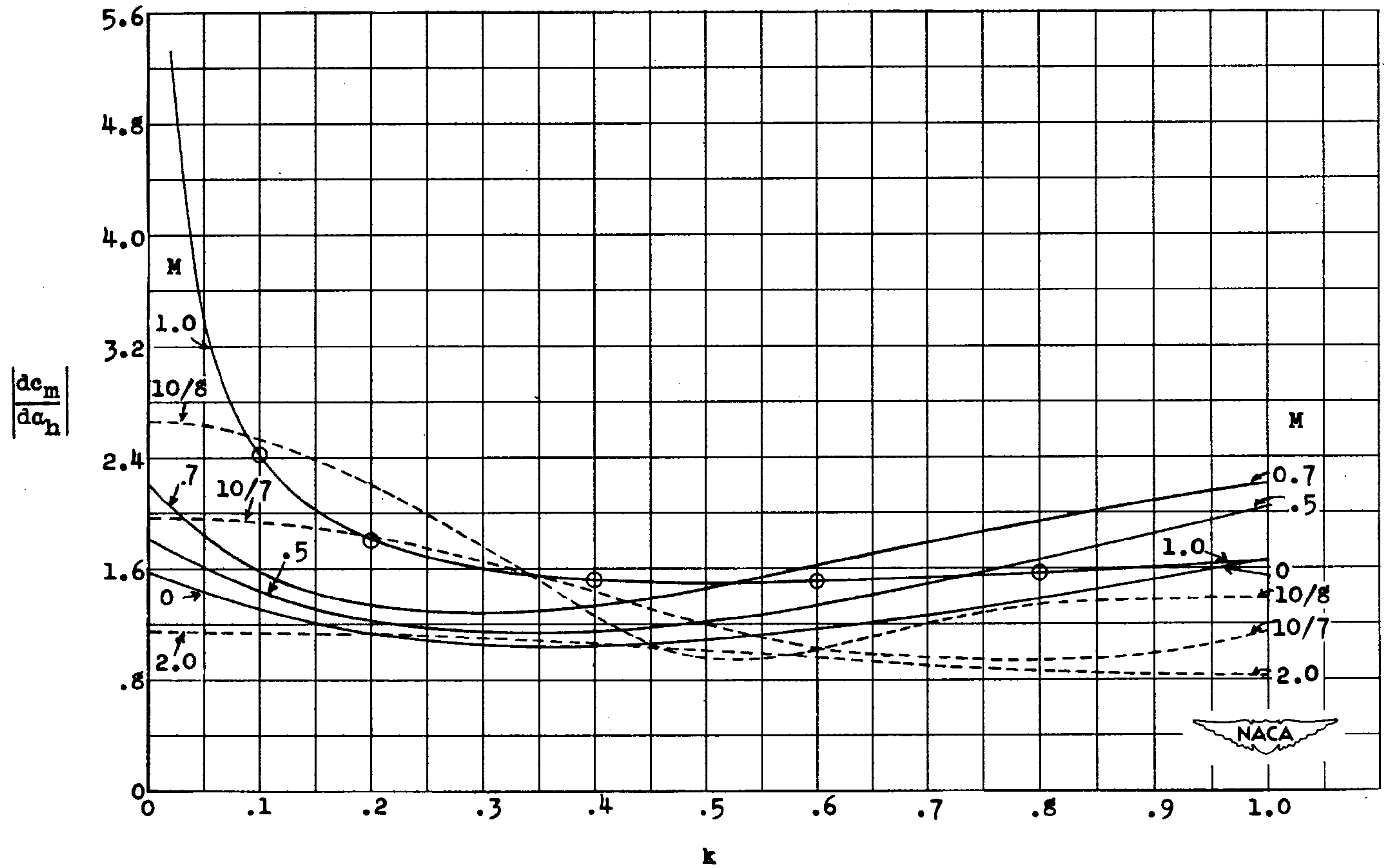
Figure 2.- Magnitude of lift-curve slope and moment-curve slope against reduced frequency for several Mach numbers.



(b) Lift-curve slope associated with pitching of wing.

$$\left|\frac{dc_l}{d\alpha}\right| = 4k^2\sqrt{L_3'^2 + L_4'^2}.$$

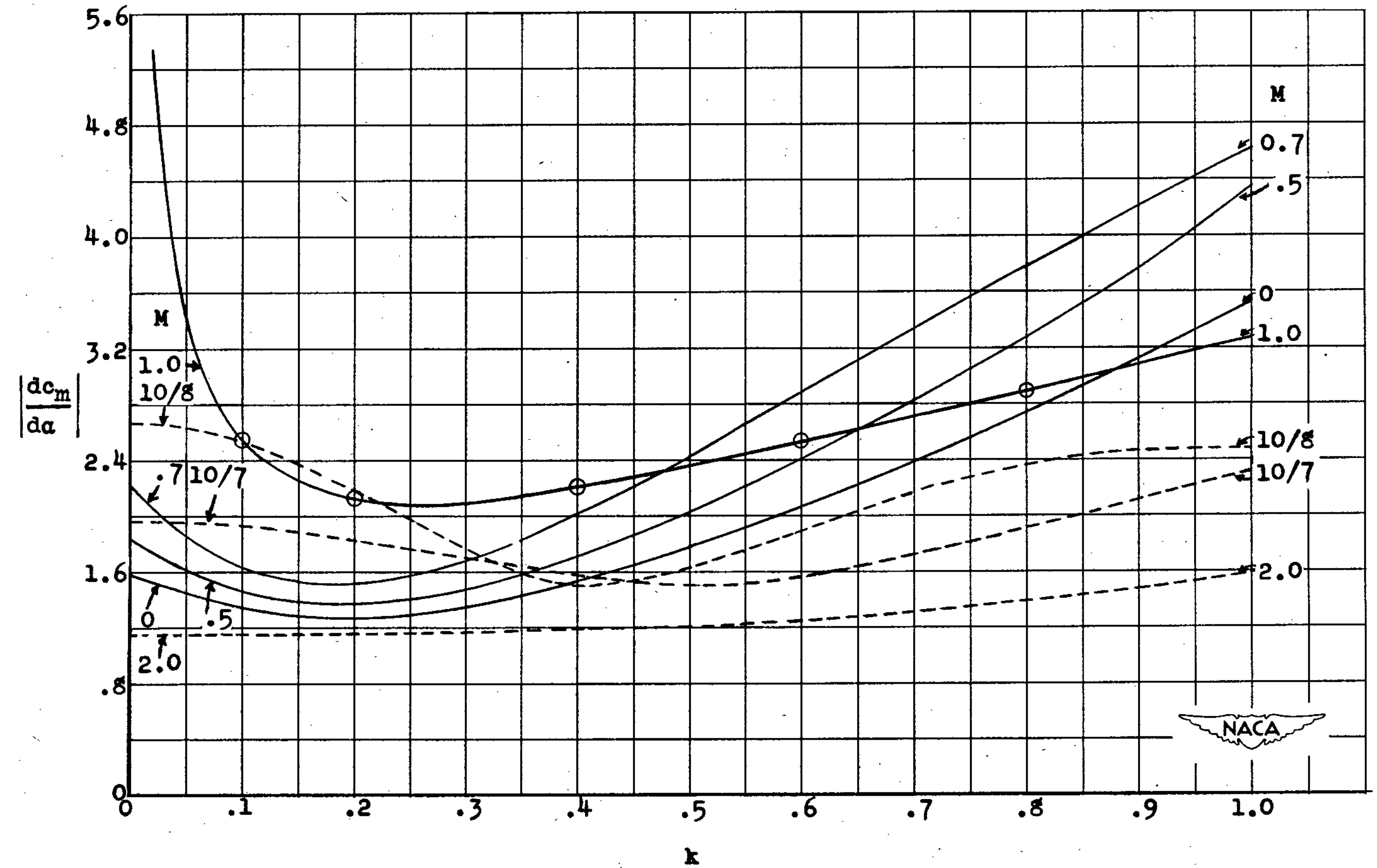
Figure 2.- Continued.



(c) Moment-curve slope associated with vertical translation.

$$\left|\frac{dc_m}{d\alpha_h}\right| = 2k\sqrt{M_1'^2 + M_2'^2}.$$

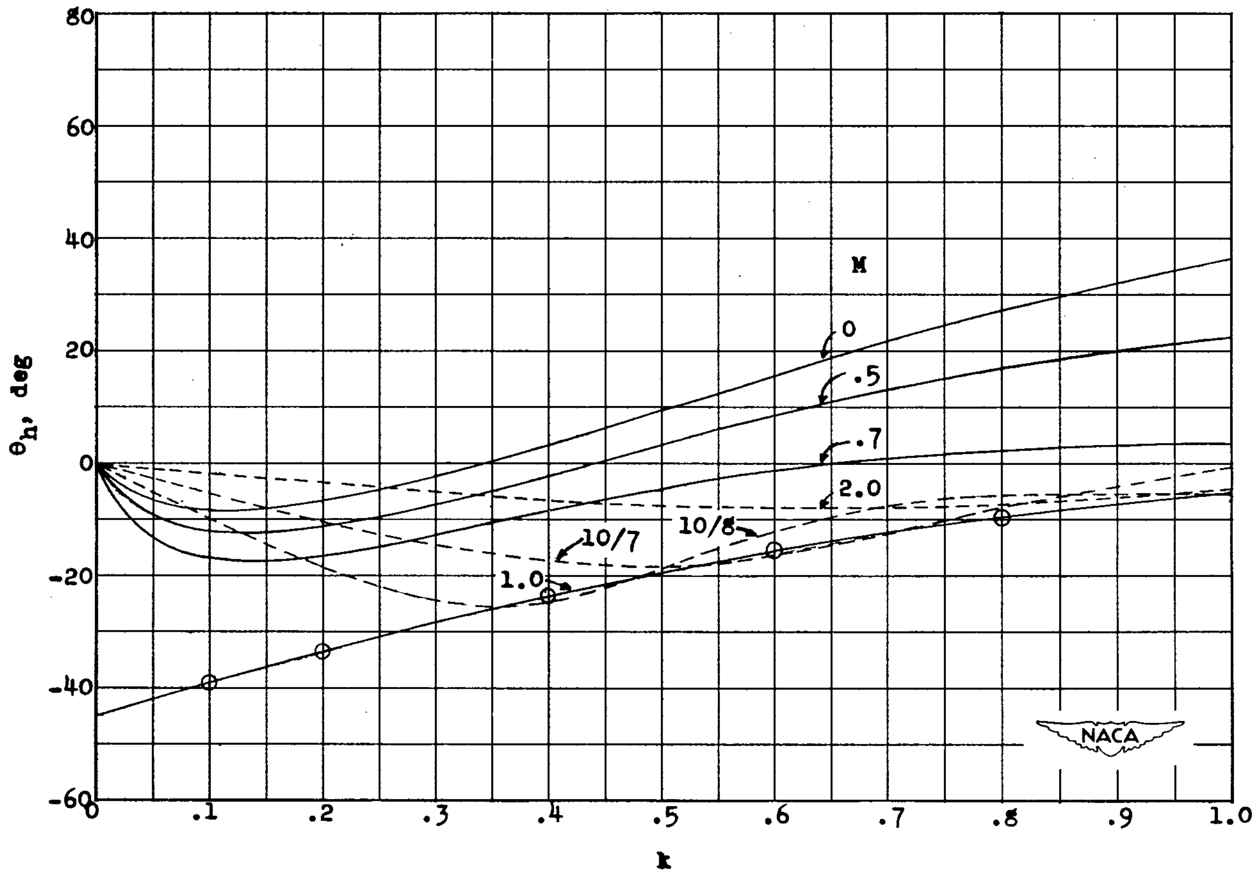
Figure 2.- Continued.



(d) Moment-curve slope associated with pitching of wing.

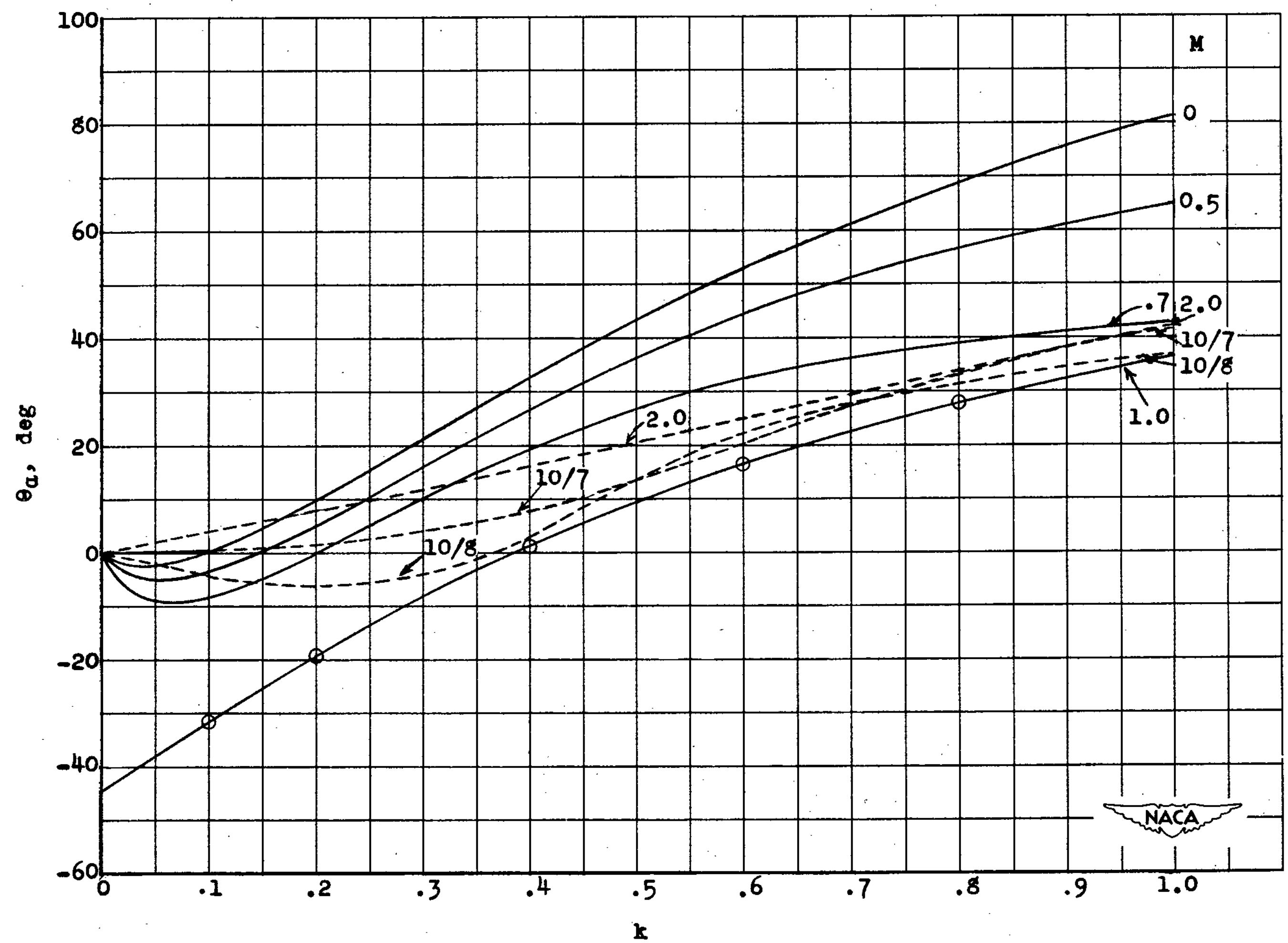
$$\left|\frac{dc_m}{d\alpha}\right| = 2k^2\sqrt{M_3'^2 + M_4'^2}.$$

Figure 2.- Concluded.



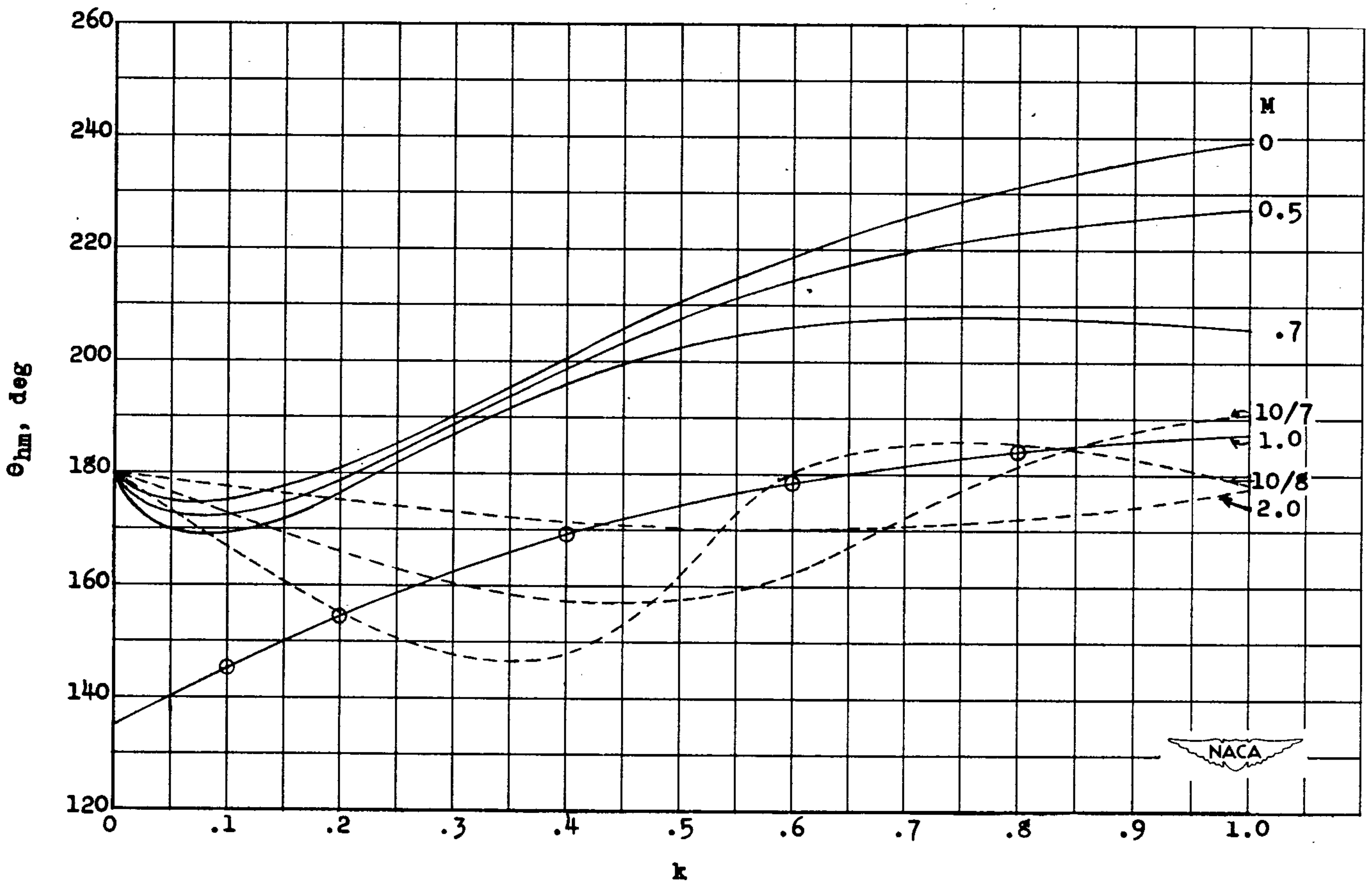
(a) Phase angle between lift vector due to vertical translation and vertical velocity vector $\dot{h}$.

Figure 3.- Phase angles plotted against reduced frequency for several Mach numbers.



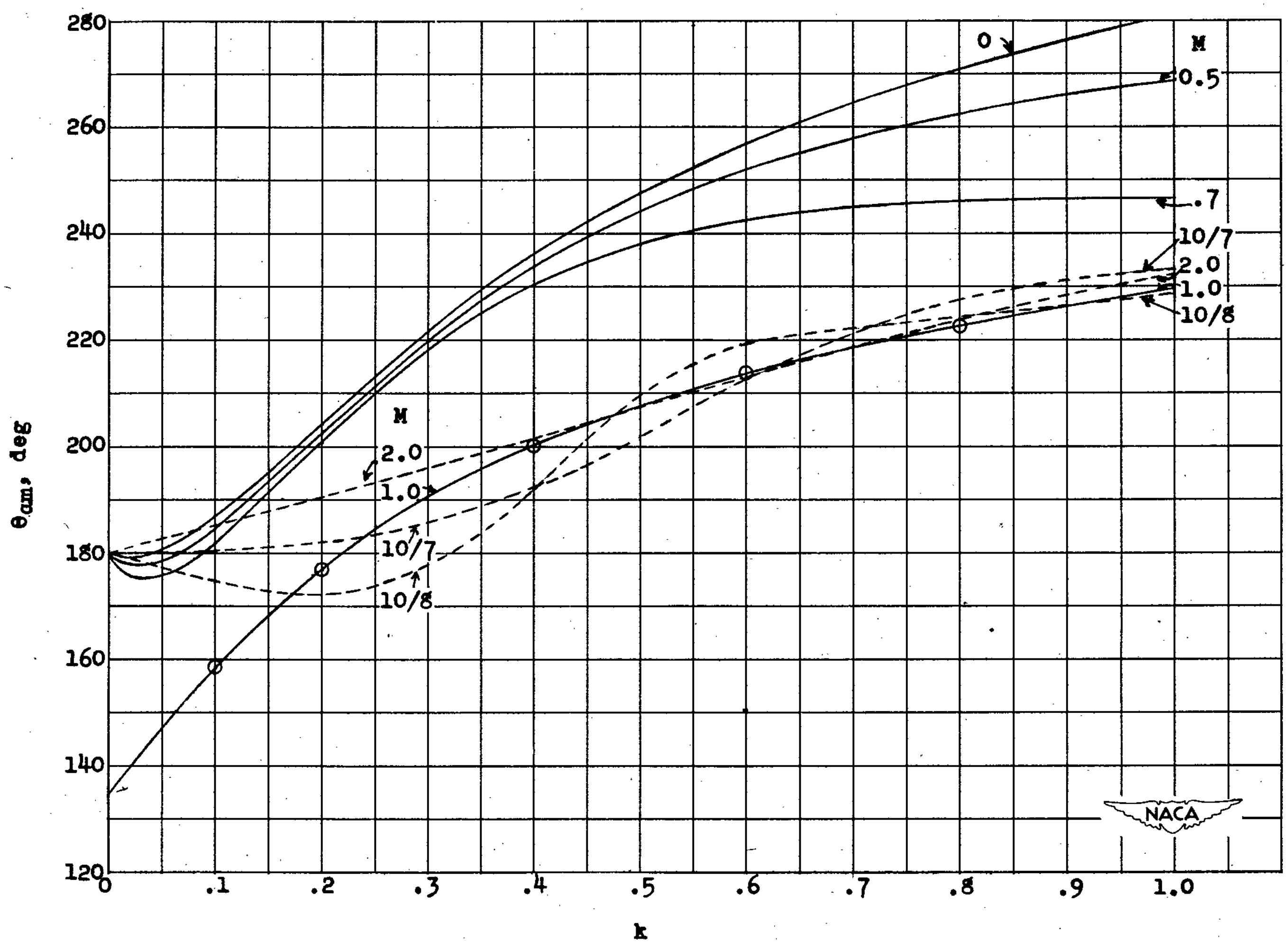
(b) Phase angle between lift vector due to pitching and angular displacement vector α.

Figure 3.- Continued.



(c) Phase angle between moment vector due to vertical translation and vertical velocity vector $\dot{h}$.

Figure 3.- Continued.



(d) Phase angle between moment vector due to pitching and angular displacement vector α.

Figure 3.- Concluded.

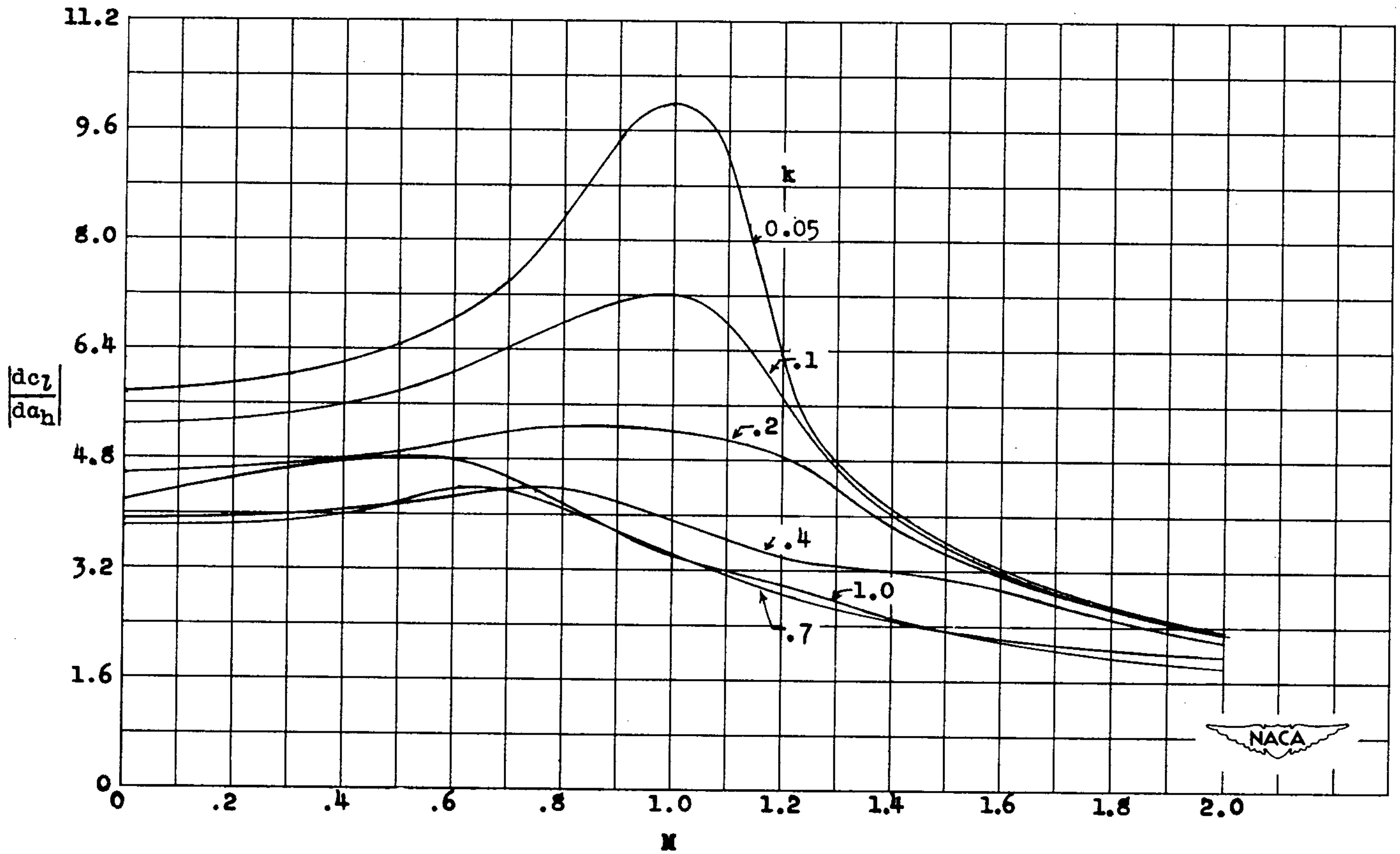


Figure 4.- Magnitude of lift-curve slope associated with vertical translation of wing against Mach number for several values of reduced frequency. $\left|\frac{dc_l}{d\alpha_h}\right| = 4k\sqrt{L_1'^2 + L_2'^2}$.

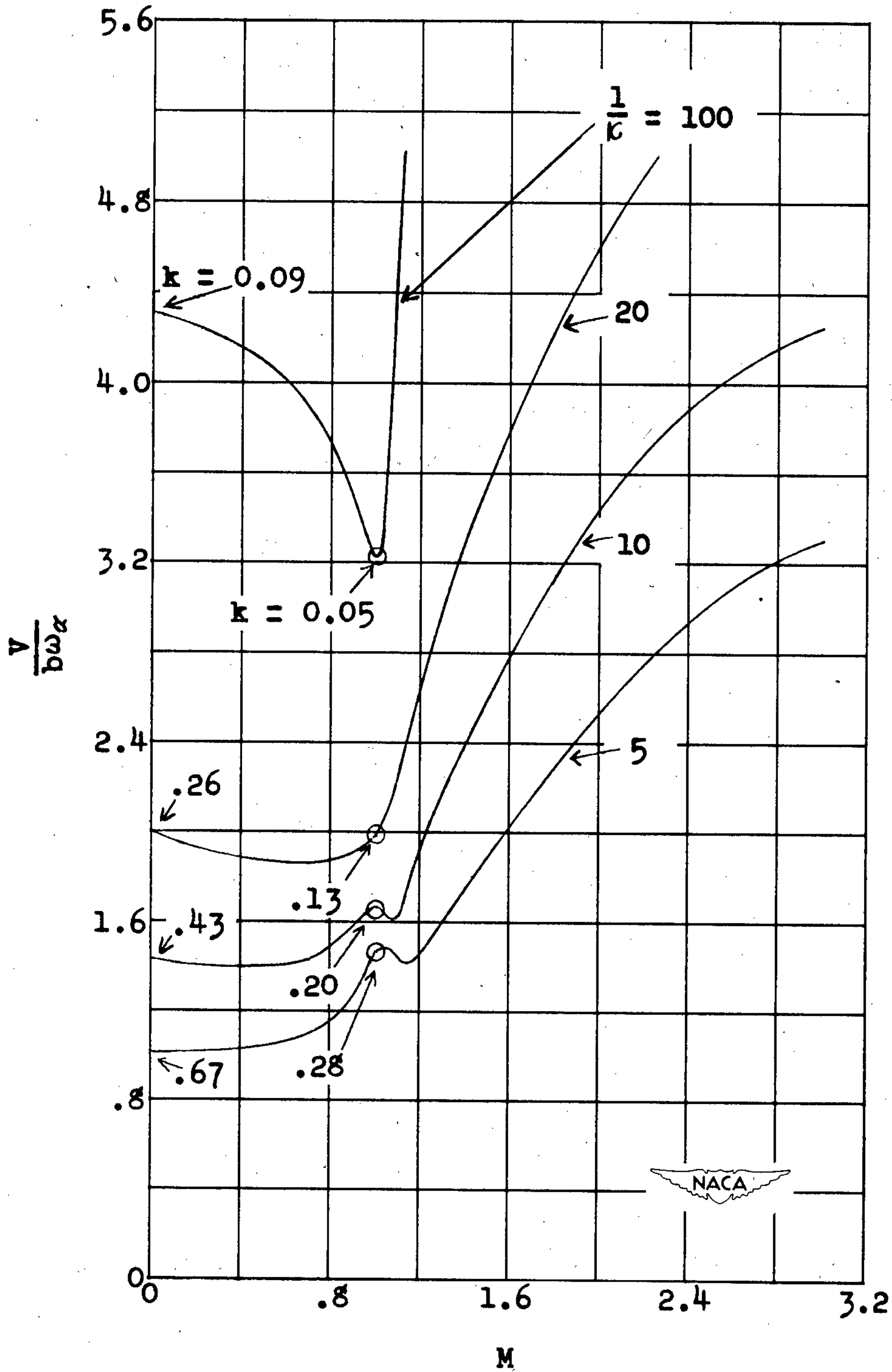


Figure 5.- Flutter-speed coefficient against Mach number for several values of $1/\kappa$ when $\frac{\omega_h}{\omega_\alpha} = 0$, $x_\alpha = 0.2$, and $a = 0$. (Figure 18 of reference 5 modified to include calculated values indicated by circles.)

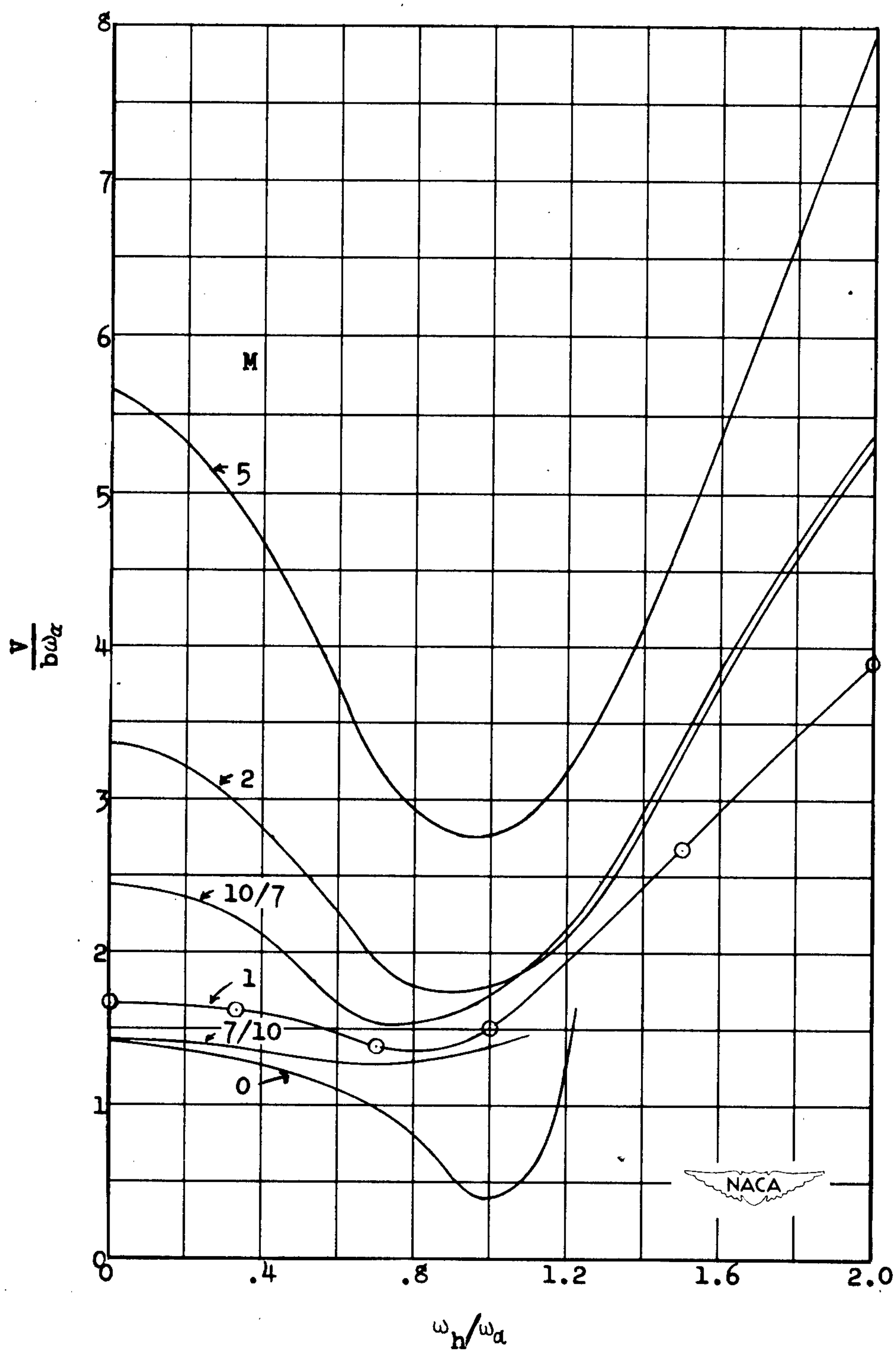


Figure 6.-Flutter-speed coefficient against frequency ratio for several values of M when $a = 0$, $x_\alpha = 0.2$, and $\frac{1}{\kappa} = 10$. (Figure 19 of reference 5 modified to include calculated values indicated by circles.)

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NACA TN 2590
National Advisory Committee for Aeronautics.
CALCULATIONS ON THE FORCES AND MOMENTS FOR AN OSCILLATING WING-AILERON COMBINATION IN TWO-DIMENSIONAL POTENTIAL FLOW AT SONIC SPEED. Herbert C. Nelson and Julian H. Berman. January 1952. 36p. diagrs., 2 tabs. (NACA TN 2590)

The linearized theory for compressible unsteady flow is used, as suggested in recent contributions to the subject, to obtain the velocity potential and the lift and moment for a thin, harmonically oscillating, two-dimensional wing-aileron combination moving at sonic speed. The velocity potential is derived by considering the sonic case as the limit of the linearized supersonic theory. The paper provides extensive tables of numerical values for the coefficients contained in the expressions for lift and moment, for various values of the reduced frequency

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$k(0 < k \leqq 3.5)$ and aileron hinge position (from 10 to 90 percent of the wing chord).

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$k(0 < k \leqq 3.5)$ and aileron hinge position (from 10 to 90 percent of the wing chord).

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